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No. 5

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Percy Morris, Holmwood, Camborne Road, Sutton, Surrey, was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Zia Uddin Ahmad, B.A., Isaac Newton Student in the University of Cambridge, Trinity College, Cambridge (proposed by E. T. Whittaker);

Daniel Buckney, 61 Strand, W.C. (proposed by W. H. M. Christie); and

Robert James Wallace, Yerkes Observatory, Williams Bay, Wis., U.S.A. (proposed by S. W. Burnham).

Eighty-five presents were announced as having been received since the last meeting, including, amongst others :—

Bonn Observatory, Veröffentlichungen : C. Mönnichmeyer, Beobachtungen der internationalen Polhöhensterne (presented by the Observatory); Cape Observatory Annals : Revision of the Cape Photographic Durchmusterung (presented by the Observatory); J. H. Schroeter, Beiträge zu den neuesten astronomischen Beobachtungen, 1788 (presented by Dr. Dreyer); Yerkes Observatory Publications, vol. 3; The Rumford Spectroheliograph

(presented by the Observatory); Portrait of Bishop Brinkley (presented by Sir R. Ball); Portrait of Manuel Johnson (presented by Dr. Rambaut);

And, in addition to the above, a series of photographs presented by the Director of the Yerkes Observatory which had been shown at the annual meeting—viz. 11 transparencies from Professor Hale's spectroheliographs of the solar surface, and 39 paper prints from photographs of stellar spectra, the Sun, instruments, &c.

Note on the Instrumental Errors affecting Observations of the Moon. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In the November number of the *Notices* I described a method of photographing the Moon among the stars as a way of obtaining its place which seemed to be freer than the methods now in use from certain systematic errors, and the following sentence occurs in the first paragraph :—

It was to obtain observations of the Moon in the first and last quarters that Airy set up the altazimuth at Greenwich in 1847; and unfortunately the instrument did not satisfactorily solve the problem.

At the time of writing these words it occurred to me that the evidence for or against the value of the altazimuth observations had never, so far as I know, been definitely published. Disparaging criticisms of a general nature have been made more than once, and were at any rate so familiar to me that the above sentence suggested itself naturally. But at the same time, for my own satisfaction, it seemed desirable to make some investigation of the behaviour of the instrument; and as an involuntary pause in our work for the *Astrographic Catalogue* set a computer free, an inquiry was started.

2. Soon afterwards I learnt that Mr. Cowell was at work on the Greenwich lunar observations, and it seemed probable that we were merely duplicating his work. But in reply to a definite question I learnt that he was not discussing the altazimuth observations at all—perhaps another illustration of the general disrepute into which they have fallen. Accordingly the inquiry was not interrupted; and it now seems fortunate that it was started, for, unless I am mistaken, the results indicate that the instrumental errors of the transit circle observations may be much larger than Mr. Cowell appears to have allowed for, and are liable to modify some of the deductions which he has extracted with such skill from the *Greenwich Observations*. In view of the fact that he is contemplating further work of the same kind, it seems important to draw attention at once to the possibilities.

3. In the Appendix to vol. 1. of the *Monthly Notices* the Greenwich observations, with both transit circle and altazimuth, from 1847 to 1861 are compared with Hansen's Tables. In discussing these observations from the point of view of theory, it is important to make certain corrections to the tables, as Mr. Cowell has done (see *Monthly Notices*, lxiv. p. 85); but my immediate concern was with the difference between transit circle and altazimuth observations—both made practically at the same time and compared with the same tables, so that any error of the tables does not matter, and was neglected. The only *instrumental* inequality of the Moon is that which runs its period with the lunation, though the season of the year must also be taken into account. Thus the observations were grouped according to the day of the lunation, counting full moon as 15; and all the lunations with full moon in January were taken together; all those with full moon in February, and so on. Blank days were filled in by simple interpolation between the dates which had observations. Sometimes this process had to be carried to an extent which might be condemned; but it does not seem likely that the general results shown below can be essentially incorrect. Then the means were taken for all the January lunations for each day, then for all the February lunations, and so on. Finally, taking the mean value for the five days 13, 14, 15, 16, 17 near full moon in January, this was subtracted from all the January days; the mean for five days near full moon in February from all the February days, and so on.

4. Assuming the tables correct, we should then have the *instrumental* error for each day compared with that at full moon, for both transit circle and altazimuth, the observations with the two instruments being of course kept separate. If the tables are not correct, the tabular errors will be common to both series; and the *differences* should give merely the differences of instrumental errors.

5. Now what should these differences be? Before full moon the first limb is observed with both instruments; after full moon the second limb.* The exceptions are confined entirely to the three days numbered 14, 15, 16, and are so few that they may be probably neglected for the purposes of the present inquiry. We should be prepared for a different apparent diameter for the two instruments, which would mean a constant difference between them, changing sign at day 15. This corresponds to the assumption made by Mr. Cowell for the transit circle observations. He says (*Monthly Notices*, lxiv. p. 96):—

The hypothesis that the error of semi-diameter is a constant

* It might at first sight appear that a complication is introduced into the altazimuth observations, because the error in longitude is deduced from errors in azimuth and Z.D.; and in Z.D. sometimes the upper and sometimes the lower limb is observed. But on the hypothesis of a simple error in semi-diameter, the effect on the longitude will in all cases be reversed after full moon.

is the best that I can make. . . . The mean correction for different ages of the Moon is no doubt a function of the age of the Moon and not a constant. If there were any means of obtaining its numerical values they could be analysed into the form

$$f(D) + \alpha \sin D + \beta \sin 2D,$$

when α , β would be corrections applicable to the results of this paper. . . . I do not think that α can possibly be as large as $0''\cdot30$, and it is possibly a good deal smaller.

6. From the point of view of the paragraph just quoted the comparison of the transit circle and altazimuth observations throws light on the possible values of α and β , though it does not determine them. If α and β are small for the transit circle they should be smaller for the altazimuth, since the observations made with the latter are made far more often in a dark sky. If the differences mentioned in § 4 are *not* constant we cannot feel so sure that α and β are small.

7. These differences are shown in the following table, wherein the separate results for both altazimuth and transit circle are also given, though these are made up of instrumental errors combined with errors of theory in a manner which need not concern us at present. In the first instance the means of all the months have been taken.

TABLE I.

Mean Error in Longitude of the Moon's Centre as compared with that at full Moon for all seasons.

Day of Luna- tion.	Altaz. 1847-1850.	Altaz. 1847-1861.	T.C. 1847-1861.	T.C.—A.		
				Uncorr.	Corr. I.	Corr. II.
2	...	+2'1	"	...	...	...
3	...	+1'2	...	...	...	...
4	...	-0'1	-0'2	-0'1	+0'9	+0'7
5	-0'1	-0'3	-1'2	-0'9	+0'1	-0'1
6	-0'5	-0'5	-2'2	-1'7	-0'7	-0'9
7	+0'1	-0'9	-2'3	-1'4	-0'4	-0'6
8	-1'2	-1'2	-2'3	-1'1	-0'1	-0'3
9	-1'7	-1'2	-2'0	-0'8	+0'2	0'0
10	-1'4	-1'1	-1'9	-0'8	+0'2	0'0
11	-1'4	-1'2	-1'8	-0'6	+0'4	+0'2
12	-0'5	-1'0	-1'7	-0'7	+0'3	+0'1
13	-0'3	-0'5	-1'4	-0'9	+0'1	-0'1
14	0'0	-0'1	-0'9	-0'8	+0'2	0'0
15	+0'5	+0'4	0'0	-0'4	...	...
16	-0'8	-0'2	+0'9	+1'1	-0'2	0'0

Day of Luna- tion.	Altaz. 1847-1850.	Altaz. 1847-1861.	T.O. 1847-1861.	T.O.—A.		
				Uncorr.	Corr. I.	Corr. II.
17	+0.5	+0.4	+1.4	+1.0	-0.3	-0.1
18	+0.3	+0.5	+1.8	+1.3	0.0	+0.2
19	+0.5	+0.5	+1.9	+1.4	+0.1	+0.3
20	+0.9	+0.7	+1.5	+0.8	-0.5	-0.3
21	+0.6	+0.9	+2.3	+1.4	+0.1	+0.3
22	+1.4	+1.1	+2.5	+1.4	+0.1	+0.3
23	+0.8	+1.0	+2.4	+1.4	+0.1	+0.3
24	+0.7	+0.8	+2.2	+1.4	+0.1	+0.3
25	-0.5	-0.2	+1.7	+1.9	+0.6	+0.8
26	...	-0.5	...	...	...	...
27	...	-0.2	...	...	...	...

8. In the first column is given the day of the lunation counting full moon in all cases as 15; to the second column reference will be made later in the paper; in the next two columns the mean errors with the different instruments. In the fifth column the simple difference is shown. Now, on the hypothesis that a different value of the semi-diameter is applicable to the two instruments, *and that only*, we may take the mean of the differences before full moon and the mean of those after to determine this difference. On the other hand, if we allow for a progressive change as well, we may establish continuity by taking the means of days close to full moon only—say three days before and three days after. The means on these two suppositions are as follows:

TC—A	Hypothesis I.		Hypothesis II.	
	10 days before.	10 days after.	3 days before.	3 days after.
	-1.0	+1.3	-0.8	+1.1

9. We notice that on either hypothesis there is a sensible divergence ($0''.3$) from symmetry. No probable explanation of this difference occurs to me at present, but it is worthy of note that we here have a curious and unexpected instrumental difference half as large as the value allowed as possible by Mr. Cowell for α , as quoted in § 5.

10. Applying these corrections for semi-diameter we get the columns shown as corr._I and corr._{II}. Whichever column we take it seems improbable that the value of α can be so small as $0''.3$; and we are here dealing with the *difference* between two instruments. The effect for either may be much larger. On the other hand it is possible that the whole error is in the altazimuth observations, and the transit circle observations are comparatively correct. But this should not be assumed without good reason being shown; for, as already remarked, the alt-

azimuth observations, rough though they may be, are made under more constant conditions than the meridian observations.

11. To check the above results it was determined to collect the information in an independent manner as follows. Only those days were included when there was both a T.C. and an altazimuth observation, and the direct difference of the errors in longitude was formed and tabulated under the day of the lunation, as before. Any error of the tables was thus eliminated by a more direct process, and the necessity of filling in blank days was avoided. Further, the observations made with the present transit circle, set up in 1851, were kept separate from those of the old one* used from 1847 to 1850. By an oversight this was not done in the previous work.

12. Let us first consider the comparisons on the hypothesis that there is simply a diameter peculiar to each instrument. Taking the means of all the comparisons before and after full, and taking groups corresponding to different seasons, we have

TABLE II. (Hypothesis I.)

	Old T.C.—Altaz. (1847-1850).		New T.C.—Altaz. (1851-1861).	
	Before Full.	After Full.	Before Full.	After Full.
Nov., Dec., Jan.	-1'6 ₃₈	+1'8 ₄₈	-0'7	+2'2
Feb., Mar., Apr.	-1'9 ₃₅	+0'7 ₃₀	-1'7	+2'6
May, June, July	-1'9 ₆₇	+0'1 ₄₅	-0'4	+2'3
Aug., Sept., Oct.	-1'3 ₅₃	+0'7 ₅₉	-1'7	+2'3
Mean	-1'6	+0'8	-1'1	+2'4

13. We see that the curious asymmetry is much accentuated now that the comparison is made in this new way. Further, that it is quite different in character for the two periods 1847-1850 and 1851-1861. The mean error is negative for the former period (-0''₄) and positive for the latter (+0''₆); which seems to imply that, unless there was a corresponding change in the altazimuth observations themselves, the mean places of the Moon found by the new transit circle differ systematically by +1''₀ in longitude from those found by the older instrument. It is of course quite possible that the change was in the altazimuth and due to

(a) *Instrumental causes*, such as the introduction of the chronographic method of registration. Even such changes as are mentioned in the volume for 1851, viz. a change in the method of illuminating the divisions or of mounting the object glass, may be responsible.

(b) *Personal*. A remarkable personality in the case of Mr. Hugh Breen was detected almost as soon as the altazimuth was erected. That observer had a lunar personality (applying to both limbs of the Moon) of 0^s₃₁. But this was detected in 1848 and the corrections made for existing observations, and Mr. Breen

* The term "old T.C." is used for brevity to denote the combination of transit instrument and mural-circle which was in use before 1851.

ceased forthwith to observe with the instrument. We may infer that such personalities were looked for with some care and probably eliminated; but some may have been overlooked.

But in either case, if the change was really in the altazimuth observations, it ought to be possible to prove this and free the transit circle observations from the suspicion of such a grave discontinuity.

14. To compare the results of Table II. with those previously found from Table I. we must combine them with the approximate weights 1 to 3; since the first series extended over three and a half years only and the second over eleven. We should then get $-1''.2$ before full and $+2''.0$ after, as compared with $-1''.0$ before and $+1''.3$ after (§ 8, Hypothesis I.) Our first results were therefore numerically too small as compared with the second. I feel tolerably sure that this is due to the way in which the first results are affected by the imperfection of the record, though I have not been able to establish this point completely. One way in which this imperfection tends to diminish these numbers is as follows:—Blank days were filled up by simple interpolation. Now, suppose we had observations on days 12 and 16. The former is subject to a correction $+1''.2$, say, and the latter to a correction $-2''.0$. On interpolation for day 13 we get a result subject to a correction $+1''.2 - \frac{1}{4}(3''.2)$ instead of $+1''.2$, as it should be; *i.e.* $0''.8$ too small, and for day 14 $+1''.2 - \frac{1}{3}(3''.2)$, or $1''.1$ too small. Any days near full moon are thus liable to dilution in this way. In fact the process of §§ 7–10 is sensibly defective for our present purpose, and is to be regarded as superseded by the more direct method now being considered. The disadvantages of the present method, which prevented its use in the first instance, are of course (1) that a good many observations are lost; but this is not serious; (2) that we get no information as to the behaviour of the altazimuth in the part of the lunation where it is specially valuable, *i.e.* when the transit circle observations fail.

15. If now, instead of including *all* the results “before full,” we limit ourselves to the three days, 12, 13, 14, as in Hypothesis II. of § 8, and similarly for “after full,” we get

TABLE III. (Hypothesis II.)

	Old T.C.—Altaz. (1847–1850).		New T.C.—Altaz. (1851–1861).	
	Before Full.	After Full.	Before Full.	After Full.
Nov., Dec., Jan....	$-1''.7_{13}$	$+2''.5_{15}$	$-0''.5$	$+0''.9$
Feb., Mar., Apr.	$-2''.5_{14}$	$+1''.5_{11}$	$-0''.7$	$+2''.1$
May, June, July	$-0''.8_{26}$	$+0''.5_{17}$	$-0''.7$	$+1''.3$
Aug., Sept., Oct.	$-0''.8_{21}$	$+0''.7_{23}$	$-1''.6$	$+2''.2$
Simple Mean	$-1''.4$	$+1''.3$	$-0''.9$	$+1''.6$

16. Inspection of Tables II. and III. suggests that the effect of the seasons is comparatively small. We will therefore, in the first instance, disregard it and take all the observations through-

out the year together, as was done in Table I. The results are given in Table IV., which corresponds with the arrangement of Table I., but is to be taken as superseding it for reasons already given.

TABLE IV.

Differences between Meridian and Altazimuth Observations of the Moon's Longitude.

Day of Lunation.	Old T.C. — Altaz. (1847-1850).			New T.C. — Altaz. (1851-1861).		
	Uncorr.	Corr. I.	Corr. II.	Uncorr.	Corr. I.	Corr. II.
5	-4 ^{''} ₂₈	-2 ^{''} ₆	-2 ^{''} ₈	-0 ^{''} ₄	+0 ^{''} ₇	+0 ^{''} ₅
6	-2 ^{''} ₆ ₁₆	-1 ^{''} ₀	-1 ^{''} ₂	-2 ^{''} ₀	-0 ^{''} ₉	-1 ^{''} ₁
7	-3 ^{''} ₂ ₁₄	-1 ^{''} ₆	-1 ^{''} ₈	-1 ^{''} ₆	-0 ^{''} ₅	-0 ^{''} ₇
8	-1 ^{''} ₉ ₁₄	-0 ^{''} ₃	-0 ^{''} ₅	-1 ^{''} ₈	-0 ^{''} ₇	-0 ^{''} ₉
9	-0 ^{''} ₄ ₂₁	+1 ^{''} ₂	+1 ^{''} ₀	-1 ^{''} ₅	-0 ^{''} ₄	-0 ^{''} ₆
10	-1 ^{''} ₁ ₂₁	+0 ^{''} ₅	+0 ^{''} ₃	-0 ^{''} ₉	+0 ^{''} ₂	0 ^{''} ₀
11	-1 ^{''} ₈ ₂₄	-0 ^{''} ₂	-0 ^{''} ₄	-0 ^{''} ₆	+0 ^{''} ₅	+0 ^{''} ₃
12	-2 ^{''} ₁ ₂₂	-0 ^{''} ₅	-0 ^{''} ₇	-0 ^{''} ₈	+0 ^{''} ₃	+0 ^{''} ₁
13	-0 ^{''} ₁ ₂₇	+1 ^{''} ₅	+1 ^{''} ₃	-0 ^{''} ₈	+0 ^{''} ₃	+0 ^{''} ₁
14	-1 ^{''} ₈ ₂₅	-0 ^{''} ₂	-0 ^{''} ₄	-1 ^{''} ₀	+0 ^{''} ₁	-0 ^{''} ₁
15	-1 ^{''} ₀ ₂₅	...	...	-0 ^{''} ₅	...	...
16	+0 ^{''} ₉ ₂₁	+0 ^{''} ₁	-0 ^{''} ₄	+1 ^{''} ₇	-0 ^{''} ₇	+0 ^{''} ₁
17	+0 ^{''} ₄ ₂₆	-0 ^{''} ₄	-0 ^{''} ₉	+1 ^{''} ₅	-0 ^{''} ₉	-0 ^{''} ₁
18	+2 ^{''} ₈ ₁₈	+2 ^{''} ₀	+1 ^{''} ₅	+1 ^{''} ₈	-0 ^{''} ₆	+0 ^{''} ₂
19	+0 ^{''} ₄ ₁₉	-0 ^{''} ₄	-0 ^{''} ₉	+2 ^{''} ₇	+0 ^{''} ₃	+1 ^{''} ₁
20	-0 ^{''} ₅ ₁₈	-1 ^{''} ₃	-1 ^{''} ₈	+2 ^{''} ₁	-0 ^{''} ₃	+0 ^{''} ₅
21	+0 ^{''} ₉ ₂₆	+0 ^{''} ₁	-0 ^{''} ₄	+2 ^{''} ₃	-0 ^{''} ₁	+0 ^{''} ₇
22	+0 ^{''} ₄ ₂₂	-0 ^{''} ₄	-0 ^{''} ₉	+2 ^{''} ₇	+0 ^{''} ₃	+1 ^{''} ₁
23	+0 ^{''} ₉ ₁₅	+0 ^{''} ₁	-0 ^{''} ₄	+2 ^{''} ₆	+0 ^{''} ₂	+1 ^{''} ₀
24	+0 ^{''} ₃ ₁₀	-0 ^{''} ₅	-1 ^{''} ₀	+2 ^{''} ₀	-0 ^{''} ₄	+0 ^{''} ₄
25	-2 ^{''} ₅ ₄	-3 ^{''} ₃	-3 ^{''} ₈	+4 ^{''} ₄	+2 ^{''} ₀	+2 ^{''} ₈

17. The comparison with the old T.C. is affected with much larger accidental errors, owing to the smaller number of observations. To reduce the accidental errors a little let us assume that the errors are repeated with reversed sign after full moon. Hence we may reverse the sign of one set—say those before full moon—and take the mean. Further, we may smooth the results by taking means of every consecutive three. It will be sufficient to take the columns headed Corr. II.

TABLE V

Day.	Means of Before and After		Smoothed.		N-O.
	Old.	New.	Old.	New.	
16	0.0	+0.1	...	...	...
17	-1.1	-0.1	0.0	0.0	0.0
18	+1.1	0.0	-0.1	+0.1	+0.2
19	-0.2	+0.4	-0.1	+0.2	+0.3
20	-1.1	+0.2	-0.7	+0.4	+1.1
21	-0.7	+0.6	-0.7	+0.6	+1.3
22	-0.2	+1.0	-0.1	+0.8	+0.9
23	+0.7	+0.8	+0.2	+0.9	+0.7
24	+0.1	+0.8	+0.1	+0.9	+0.8
25	-0.5	+1.2	...	...	...

18. These results are striking. They show, either (a) that the old transit circle and the new would give very different coefficients for the parallactic inequality; or (b) that the altazimuth was capable of behaving quite differently in the years 1847-1850, and in the years 1851-1861; that is to say, *the same* instrument was capable of assigning two very different values to this inequality.

In either case it seems to be necessary to make a careful study of the instrumental errors before deducing a value for the solar parallax entitled to any weight at all.

19. It throws some light on the question whether the altazimuth observations are at fault to recur to the results tabulated in Table I. and separate the group for the dates 1847-1850. The defects of these figures have already been noticed, but they chiefly concern the observations near full moon, and, further, they would be practically common to the whole series. Hence the second column of Table I. was prepared, and by comparing it with the next column it will be seen that there is a fairly strong presumption against any essential change in the altazimuth observations.

Summary.

(a) During the years 1847-61 observations of the Moon were made at Greenwich with three different instruments, viz. the altazimuth for the whole period, the present transit circle from 1851 to 1861, and the old transit and mural circle from 1847 to 1850. The altazimuth observations can thus be compared with those made with both the other instruments. The comparison is made through the medium of Hansen's tables, from the columns H-O in the Appendix to *Monthly Notices*, vol. 1.

(b) The object of the comparison is to discuss *instrumental* errors, and not errors of theory; but it will show how far certain

terms in the lunar theory, especially the parallactic inequality, are affected by instrumental errors.

(c) The Moon's longitude determined by the old meridian instruments differs from that by the altazimuth, *after allowing for different apparent diameter*, by a *variable* quantity which rises to $-0''.7$ at about the quarters compared with full moon, and is represented in the fourth column of Table V.

(d) The new transit circle, on the other hand, shows an increasing positive difference, slowly increasing even past the quarters (see the fifth column of Table V.)

(e) The evidence is against this change being due to any change in the altazimuth.

(f) It therefore seems probable that meridian instruments may give very different values for the parallactic inequality; and the results derived by Mr. Cowell on pp. 95-98 of *Monthly Notices*, vol. lxiv., especially that for the Sun's parallax, must, until these instrumental errors have been more fully discussed, be received with caution.

University Observatory, Oxford:
1904 February 20.

Methods of Analysis of Moon's Errors and some Results.

By P. H. Cowell, M.A.

I have had a very short glance at Mr. Nevill's paper* in MS. before it was sent to the printers, and in consequence I am now giving a full account of my methods, as my recollection is that Mr. Nevill misunderstands me in a few points. I gladly take the opportunity of acknowledging the assistance that Mr. Nevill's paper is sure to be to me, and I must also admit two or three errors in my paper that Mr. Nevill points out. Nothing in my investigations at present has turned upon these errors, but it is naturally of service to me to have them pointed out. I may add that as regards the term in $\cos \varpi$ I followed Airy too rashly. He expressly states that this term is not required by theory.

My method briefly is to analyse the observed errors. I naturally use the theory to suggest periods, and as far as possible to interpret the results, but my method is essentially empirical. I hope to obtain from the observations all inequalities in the errors of Hansen's tables exceeding $0''.5$, except, perhaps, in a few instances of terms of very long period, where such accuracy is probably unattainable. A term of less than $0''.5$ will probably not be detected, but coefficients of terms examined should in

* It has been found necessary to send the proofs of Mr. Nevill's paper to the author for correction. The paper cannot therefore appear in the present Number. Professor Brown's paper is also deferred for a similar reason.

general be obtained from fifty-four years' observations correctly to 0''.1 or less. There are two essential points in such an analysis, both of them well known. The first admits of no variation in its method of treatment. The second I treat in a somewhat novel fashion, which I shall endeavour to explain.

The first of these points is the near equality of many of the periods. Let us, for example, consider the inequalities with arguments nearly equal to the mean anomaly g . Let them be denoted by $\Sigma a_s \sin(g + \theta_s)$, where the quantities θ denote angles of very long period (differing from suffix to suffix). The observations are analysed into the form for each period of analysis —

$$\text{Error} = (\text{coefficient of } \sin g) \times \sin g + (\text{coefficient of } \cos g) \times \cos g$$

It is then attempted to equate

$$(\text{coefficient of } \sin g) \text{ to } \Sigma a_s \cos \theta_s$$

and

$$(\text{coefficient of } \cos g) \text{ to } \Sigma a_s \sin \theta_s$$

The quantity θ_s is obtained approximately from the observations, and then when possible identified with some argument suggested by theory. The coefficient a_s is obtained by solving the equations.

Investigations usually proceed on these lines. Certainly Professor Newcomb's (1876), Mr. Nevill's (1884), and Mr. Bakhuyzen's (1903) do. I have departed in one small point only from the practice established by these writers. My period of analysis is 400 lunar days instead of one year. My reasons for this are: (i.) 400 lunar days is fifteen periods of the mean anomaly, fourteen of the parallactic inequality, and thirteen of the evection; it is also a round number; (ii.) my plan is to remove all new inequalities from the tabular places, and to effect this speedily I have arranged the observations in columns of forty lines each. Each line of every column corresponds to a transit of the Moon, whether observed or not. This arrangement greatly facilitates the analysis as well as the application of corrections. It obviously requires the period of analysis to contain an exact number of columns.

The second point depends upon the want of symmetry in the distribution of the observations with reference to the age of the Moon. It is evident that, if the Moon was only observed when it was full, a real inequality $a \sin \phi$ could not be distinguished from apparent inequalities $-a \sin(\phi + D)$ and $-a \sin(\phi - D)$. In my paper in the December number of the *Monthly Notices* I showed that in 5647 observations the mean value of $\cos D$ is $-2721 \div 5647 = -0.48$, and of $\sin D + 416 \div 5647 = +0.07$. The latter quantity may be neglected, but from the former I infer the rough rule that a real inequality $a \sin \phi$ will give rise to two apparent inequalities $-\frac{1}{2}a \sin(\phi + D)$ and $-\frac{1}{2}a \sin(\phi - D)$. Conversely, given an apparent inequality $\beta \sin \phi$, it is not certain

that such a term is required by the tables. The term actually required may be $-2\beta \sin(\phi + D)$ or $-2\beta \sin(\phi - D)$. The terms $\phi \pm D$ must be examined before the nature of the apparent inequality can be determined. Below, when I come to the results, a few instances will be given where a second analysis (after removing some terms) has amply justified the method of reasoning.

n observations are considered to indicate an apparent inequality $\beta \sin \phi$, where

$$\beta = (\Sigma \varepsilon \sin \phi) / \frac{1}{2}n$$

ϕ being any argument, of period short compared with the period of analysis, and not either equal, or nearly equal, to D , the age of the Moon, and ε being the observed error.

It will be seen below that ϕ has been put equal to g , the mean anomaly, $2D - g$, the evection, and subsequently g a second time after the removal of some terms, the period of analysis being 400 lunar days, and 48 such periods being used. Also ϕ has been put equal to g' , the Sun's mean anomaly, and $g - D$, an argument whose period is 400 lunar days, the whole 48×400 lunar days being in these cases taken as a single period of analysis.

In the December *Monthly Notices*, at the beginning of my paper on the semi-diameter &c., I mentioned that the tabular places from 1847 to 1901 had been corrected, when necessary, for Newcomb's corrections and for Hansen's term with wrong sign. Previous to the investigation of that paper they were also corrected for five other terms, making seven corrections in all. They have subsequently been corrected for

(8) semi-diameter (correction $0''.4$ in accordance with the results of my paper, this being a reasonable approximation to $\pm 0''.346 + 0''.120 \sin D$).

(9) The long-period terms were taken out approximately by subtracting suitable constants from all the errors throughout a whole period of analysis.

This may be looked upon as equivalent to equating each error to $c + \beta \sin \phi$, instead of $\beta \sin \phi$ only. The normal equation for β will then contain c with a small coefficient; as an approximate value for c , the mean error of the period may clearly be taken. Since in some periods of analysis c exceeds $6''$, it must be more accurate to take account of it. The error of omitting it is $\frac{2c}{n} \Sigma \sin \phi$, and $\frac{1}{n} \Sigma \sin \phi$ is liable to be about 0.1 . By this

method the error is reduced to $\frac{2\Delta c}{n} \Sigma \sin \phi$, Δc being the quantity by which the estimate of c differs from the truth. When the observations happen to be well distributed as regards the argument ϕ , $\Sigma \sin \phi$ will be nearly zero.

$$(10) \quad -0''.246 \sin(2D - g)$$

a correction to the evection.

$$(11) \quad +0''.316 \sin (g + 2\omega - 3J + 7^\circ)$$

a term due to the action of *Jupiter*.

$$(12) \quad +0''.55 \sin g'$$

a correction to the annual equation.

$$(13) \quad -0''.55 \sin (g - D),$$

a correction to Hansen's tables that I have discovered by analysis, but which is confirmed by the revised results of the *Darlegung*.

The following table gives the results of the different analyses.

Coefficients for 48 periods of analysis of 400 lunar days each of terms in the tabular minus observed errors of longitude in tenths of a second of arc.

	$\cos g.$	$\sin g.$	$\cos (2D - g).$	$\sin (2D - g).$	$\cos g.$	$\sin g.$
86	- 4	- 1	+ 4	+ 6	- 3	- 1
87	- 3	-10	+ 2	+10	- 1	- 7
88	-18	-13	0	+ 8	-15	- 4
89	-24	- 6	- 5	+ 9	-19	0
90	-14	+ 9	+ 2	+ 2	-12	+ 6
91	-18	+ 9	+ 1	+ 2	-13	+ 6
92	- 2	+ 7	- 1	+ 6	- 1	+12
93	+ 5	- 5	+ 1	+ 8	+ 5	- 1
94	+ 9	- 9	0	+11	+ 3	+ 2
95	-12	-11	- 1	+10	+ 1	- 6
96	-22	-20	- 6	+ 4	-13	- 9
97	-21	+ 2	-12	+ 6	- 9	- 2
98	-18	+ 6	- 6	- 1	-13	- 6
99	+ 4	+ 4	- 7	- 6	- 9	+ 7
100	+ 7	- 1	- 1	- 4	+ 6	+ 5
101	+ 8	0	- 3	0	+ 1	+ 2
102	- 3	- 7	+ 1	0	+ 1	- 2
103	0	- 6	+ 2	+ 1	- 1	- 6
104	- 5	-11	+ 4	+ 4	- 6	- 3
105	- 8	0	- 7	+ 6	-10	0
106	- 9	0	- 6	+ 6	- 8	+ 4
107	0	+10	- 1	+ 8	0	+ 7
108	+ 6	+13	- 2	+ 4	+ 4	+11
109	+ 5	+ 5	- 1	+ 6	+ 1	+ 5
110	+ 7	- 1	+ 2	+ 5	+ 4	- 1
111	+13	+ 1	- 7	+13	+ 9	+ 4
112	- 2	0	- 4	+12	0	0
113	- 1	+ 4	- 3	+10	- 1	- 1
114	- 3	+15	- 1	+ 2	+ 3	+ 9
115	+ 5	+17	- 4	+ 3	+ 7	+14

	$\cos g.$	$\sin g.$	$\cos (2D-g).$	$\sin (2D-g).$	$\cos g.$	$\sin g.$
116	+ 10	+ 4	- 8	+ 3	+ 10	+ 3
117	+ 6	+ 3	+ 3	0	+ 7	+ 1
118	+ 9	- 3	+ 1	+ 7	+ 11	0
119	- 9	- 15	0	+ 10	- 7	- 9
120	- 19	- 5	0	+	- 13	- 2
121	- 14	+ 3	- 2	+ 10	- 7	+ 5
122	- 11	+ 9	- 2	+ 10	- 6	+ 11
123	- 7	+ 6	- 1	+ 9	- 3	+ 7
124	+ 3	+ 8	- 7	+ 4	+ 4	+ 11
125	+ 11	- 4	- 4	- 2	+ 14	+ 1
126	+ 10	- 10	- 5	+ 9	+ 9	- 5
127	+ 3	- 15	- 7	+ 14	+ 6	- 6
128	- 6	- 13	- 6	+ 5	- 3	- 5
129	- 9	+ 1	- 3	+ 8	- 6	+ 6
130	- 2	- 4	- 6	+ 3	0	+ 3
131	+ 3	+ 4	- 4	- 3	+ 1	+ 7
132	+ 9	- 3	- 6	+ 2	+ 7	+ 6
133	+ 7	- 14	- 3	+ 5	+ 1	- 3
Means	- 0''26	- 0''10	- 0''25	+ 0''52	- 0''13	+ 0''16

The first two columns ($\cos g$ and $\sin g$) in the table were derived from the errors after correction (8) and before correction (9). They are liable therefore to moderately large accidental errors in certain periods proportional to $\Sigma \cos g$ and $\Sigma \sin g$ respectively.

The third and fourth columns ($\cos (2D-g)$ and $\sin (2D-g)$) were derived after correction (9) and before correction (10).

The fifth and sixth columns ($\cos g$ and $\sin g$) were derived after correction (13).

In addition to the above, by including all the observations into one single period of analysis, after correction (9), I have obtained the following *apparent* corrections. The significance of the word *apparent* will be evident from the introductory remarks to this paper :

$$\begin{aligned}
 &+ 0''\cdot 59 \sin (g-D) + 0''\cdot 34 \cos (g-D) \\
 &- 0''\cdot 83 \sin g' - 0''\cdot 33 \cos g' \\
 &+ 0''\cdot 40 \sin (\omega - \omega') - 0''\cdot 35 \cos (\omega - \omega')
 \end{aligned}$$

(The notation is Hansen's, with the additional symbol $D = g - g' + \omega - \omega'$.)

The coefficients of $\cos g$ and $\sin g$ (first set) have been analysed for planetary terms.

Since in the case of planetary terms there is no question of

arguments differing by D , the distinction between real and apparent corrections does not arise.

$$\text{Coefficient of } \cos g = -0''.21 \sin (2\pi - 3J + 7^\circ) \\ + 0''.37 \sin (2\pi - 2J) - 0''.17 \cos (2\pi - 2J)$$

$$\text{Coefficient of } \sin g = -0''.52 \cos (2\pi - 3J + 7^\circ) \\ + 0''.02 \sin (2\pi - 2J) + 0''.09 \cos (2\pi - 2J)$$

Combining the two terms and omitting small terms we obtain

$$(\text{from periods } 86-133) - 0''.36 \sin (g + 2\pi - 3J + 7^\circ) \\ + 0''.23 \sin (g + 2\pi - 2J)$$

In a similar manner, using the values of a and b published in the November *Monthly Notices*, we obtain

$$(\text{from periods } 1-48) \quad \quad \quad + 0''.20 \sin (g + 2\pi - 2J)$$

$$(\text{from periods } 42-89) \quad \quad \quad + 0''.28 \sin (g + 2\pi - 2J)$$

It appears therefore that the tabular places require the correction

$$+ 0''.36 \sin (g + 2\pi - 3J + 7^\circ) - 0''.23 \sin (g + 2\pi - 2J)$$

The first term agrees closely with theory, for Radau gives $+ 0''.316$ as the coefficient.

The second term indicates a correction $- 0''.23$ to the $- 0''.893$ already applied. This term is of especial interest, for it will be remembered that it was discovered empirically by Professor Newcomb in 1876, who assigned it a coefficient $1''.5$. Mr. Neison first pointed out that the term is due to the action of *Jupiter*. It appears from this paper that the coefficient given by the observations is $1''.1$. Hill and Radau both obtain $0''.9$ from theory.

Coming now to the solar terms, I obtain the following *apparent* terms in the tabular *minus* observed errors:

$$- 0''.52 \sin (g - 2D) - 0''.25 \cos (g - 2D)$$

$$+ 0''.59 \sin (g - D) + 0''.34 \cos (g - D)$$

$$- 0''.10 \sin g - 0''.26 \cos g$$

Some more terms are given below after the discussion of the above. The coefficients of $\sin g$ and $\cos g$ are from the first analysis.

The cosine terms, if real, denote an error of epoch. Clearly no change of the epoch of g will explain all three terms. The terms are, however, small. I hold over the discussion until the tabular places from 1750 to 1851 have been revised and the observations discussed.

The three sine terms suggest the real corrections

$$+0''.25 \sin(g-2D) - 0''.55 \sin(g-D) - 0''.17 \sin g$$

are required by the tabular places, for a real correction of this form will produce an apparent correction

$$\begin{aligned} & (+0''.25 + 0''.27) \sin(g-2D) \\ & (-0''.12 - 0''.55 + 0''.08) \sin(g-D) \\ & (+0''.27 - 0''.17) \sin g \end{aligned}$$

on the principles explained above, thus annihilating the terms indicated by the errors with an accuracy that is almost suspicious.

It will be noticed that the terms $+0''.25 \sin(g-2D) - 0''.55 \sin(g-D)$ have been applied, and the second analysis gives for the tabular minus observed errors $+0''.17 \sin g$. This second analysis bears out in a remarkable degree the substantial accuracy of the assumption that a real correction $\beta \sin \phi$ produces apparent corrections

$$\beta \sin \phi - \frac{1}{2} \beta \sin(\phi + D) - \frac{1}{2} \beta \sin(\phi - D)$$

The explanation of these corrections is interesting. The coefficient of the evection in Hansen's tables is $4586''.690$ (see January *Monthly Notices*); the true coefficient is therefore $4586''.44$. The theoretical coefficient for an elliptic coefficient $22639''.5$ is about $4586''.34$. The difference $0''.10$ is either accidental or points to an ellipticity in the figure of the Moon only one-sixth of what Hansen assumed. It may be recalled that my paper in the December *Monthly Notices* on the evidence of the variation tended to support Hansen's hypothesis; but there can now be little doubt that the ellipticity of the Moon is insensible or nearly so, and that the apparent increase of the variation is due to the conditions of observation, as Professor Newcomb has pointed out.

I did not expect to find so large a correction as $-0''.55 \sin(g-D)$ in a term where Hansen's *Darlegung* and Delaunay are in such close accordance. The coefficients given by these two authorities are respectively $18''.54$ and $18''.60$ when reduced to a parallax $8''.76$, and when Delaunay's series is completed by induction. The difficulty is explained, however, by the fact that Hansen's tables differ considerably from the *Darlegung* in this term. The coefficient in the tables is $19''.15$, and the correction $-0''.55$ might therefore have been anticipated.

The value of the principal elliptic coefficient now appears to be

$$22640''.17 - 0''.55 - 0''.17 = 22639''.45$$

on the assumption that the terms in $g \pm D$ are

$$-8''.54 \sin(g+D) + 18''.60 \sin(g-D).$$

As, however, the former term apparently should be

$$-8''.46 \sin (g+D)$$

the principal elliptic coefficient should be

$$22639''.49$$

My preliminary discussion of Airy's period 1750-1851 if reduced to the same terms

$$-8''.46 \sin (g+D) + 18''.60 \sin (g-D)$$

gives $22639''.53$

a substantial agreement. The result, however, cannot be considered definite until Professor Brown has determined the two "allied" coefficients.

It will be noticed that all the coefficients given in the principal table of this paper show a periodicity in eight periods of analysis or nine years. This period is to be identified with $\varpi - \varpi'$ or $\omega - \omega'$. Solving on this assumption, I obtain for the tabular *minus* observed errors

$$+0''.24 \sin (g-2D+\omega-\omega')$$

$$-0''.83 \sin (g-D+\omega-\omega') - 0''.33 \cos (g-D+\omega-\omega')$$

$$+0''.81 \sin (g+\omega-\omega') + 0''.46 \cos (g+\omega-\omega')$$

the last line referring to the first analysis for g after correction (8). The argument of the second line is g' , the Sun's mean anomaly.

The above apparent corrections seem to point to true corrections

$$+0''.55 \sin g'$$

$$-0''.57 \sin (g+\omega-\omega') - 0''.46 \cos (g+\omega-\omega')$$

being required.

It will be seen that I have applied $+0''.55 \sin g'$. As far as I can make out only half this term is not required by theory, but perhaps Professor Brown's theory when completed will verify the whole. This at present seems unlikely; but as to the reality of the term, I consider the evidence immensely strengthened by the corroboration derived from the "allied" term $g'-D$.

I have not applied corrections

$$-0''.57 \sin (g+\omega-\omega') - 0''.46 \cos (g+\omega-\omega')$$

because of the extreme uncertainty of their real nature. No cosine term is required by theory, and the coefficient of the sine

term appears to require theoretically an alteration of only $0''.27$. This suggests that for the outstanding part the period may not be $g + \omega - \omega'$, but some slightly different period. Two hypotheses have occurred to me: one is that the argument should be $g + \varpi$ the Moon's longitude, or that the term is due to systematic errors of observation (since ϖ' is practically a constant, analysis of the errors cannot possibly distinguish between $g + \varpi$ and $g + \varpi - \varpi'$); the other hypothesis is that the argument is $g + 2\varpi + 3V - 5E$, and that there is something wrong with the theoretical coefficient of the *Venus* term. The argument $g + 2\varpi + 3V - 5E$ moves more slowly than the argument $g + \omega - \omega'$ by $4^\circ.22$ in 400 lunar days, or by one revolution in a century. Hence, fifty years' observations cannot distinguish between the two arguments, and the point must be held over until the Airy period 1750-1851 is re-discussed.

A comparison of my January paper with Professor Newcomb's transformation of Hansen's theory and comparison with Delaunay shows that the following corrections are still required in the tabular places:

$$\begin{aligned}
 &-0''.27 \sin (g + \omega - \omega') \\
 &+0.40 \sin (g - g') \\
 &-0.45 \sin (g + 2g') \\
 &-0.29 \sin (g' - 2\omega + 2\omega') \\
 &+0.27 \sin (g - g' + 2\omega - 2\omega') \\
 &-0.26 \sin (g + 2\omega) \\
 &-0.72 \sin (\omega - \omega') \\
 &-0.22 \sin (g + g' - \omega + \omega') \\
 &-0.32 \sin (2g - 3g' + 4\omega - 4\omega')
 \end{aligned}$$

The first term has just been alluded to; the coefficient of the seventh term seems too large, judging from the observations; as to the sixth term I have estimated that corrections

$$-0''.14 \sin (g + 2\omega) + 0''.12 \cos (g + 2\omega)$$

are required by the observations. The agreement is not very close, but the analysis can only be approximate in view of the large movement of 2ω in one period of analysis. The fourth term is "allied" to the argument $g - \omega + \omega'$, and at present my analysis throws no light upon any of the other terms.

I conclude this paper with giving the necessary data for identifying my periods of analysis with the calendar, and for obtaining the values of the arguments at various epochs. The last two lines refer to Hansen's empirical *Venus* terms.

Values of various Arguments in the middle of the 44th period of analysis and their movements in 400 lunar days or one period of analysis.

Argument. Calendar	Value. 1800 Jan. 3 ^d 6 G.M.T.	Movement. 414.02 Solar Days.
g	159°7658	5409°1681
g'	4°1375	408°0583
ω	192°7447	68°0478
ω'	246°4312	21°9435
Ω	33°0749	-21°9241
D	101°9418	5047°2141
$g-D$	57°8240	361°9540
$\varpi = \omega + \Omega$	225°8196	46°1237
$\varpi' = \omega' + \Omega$	279°5061	0°0194
$\varpi - \varpi' = \omega - \omega'$	306°3135	46°1043
Venus V	151°2198	663°3141
Earth E	103°6435	408°0619
Mars M	234°5551	216°9601
Jupiter J	82°1455	34°4014
$2\varpi - 2J$	287°3482	23°4445
$2\varpi - 3J + 7^\circ$	212°2027	-10°9570
$2\varpi + 3V - 5E$	27°0811	41°8802
$A + 30^\circ$	213°8946	1°4953
$8V - 13E + 274^\circ 14'$	136°6262	1°7081

Comparisons of the Geocentric Places of the Sun and Major Planets calculated from the Tables of the American Ephemeris Office, with their Places calculated from Le Verrier's Tables, for the year 1906. By A. M. W. Downing, D.Sc., F.R.S.

This paper is supplementary to those published in *Monthly Notices*, vol. lix. No. 10, and vol. lxii. No. 1.

The American Tables of the Sun, *Mercury*, *Venus*, *Mars*, *Uranus*, and *Neptune* are due to Professor Newcomb; those of *Jupiter* and *Saturn* to Mr. G. W. Hill. These Tables are adopted in calculating the ephemerides given in the *Nautical Almanac* for 1906. The corresponding places in the *Connaissance des Temps* for the same year are from Le Verrier's Tables.

The comparisons have been made by reducing the quantities given in the *Nautical Almanac* from Greenwich noon to Paris noon at convenient intervals throughout the year, and then finding the differences between the reduced quantities and the

corresponding quantities in the *Connaissance des Temps*. For the right ascensions the discordances are given in arc of a great circle, as well as in time, to render them comparable with the discordances in declination.

The abnormal discordances in the case of *Venus*, near the time of inferior conjunction with the Sun in 1906, are noteworthy as appearing to show that the ordinary methods of computation are not sufficiently refined to ensure accuracy in the geocentric places corresponding to these critical positions of the planet in its orbit.

With regard to *Saturn* it will be remembered that M. Gaillot has succeeded in rectifying Le Verrier's theory of this planet, which has been found to be defective in certain particulars. The publication of the corresponding corrections to the Tables will be awaited with interest.

SUN, 1906.—Corrections to Le Verrier's Tables.

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
Jan. 2	-07	-10	-04	July 5	-01	-01	+03
10	-06	-08	-03	13	00	00	+02
18	-04	-06	-04	21	-03	-04	+02
26	-04	-06	-04	29	-03	-04	+04
Feb. 3	-04	-06	-04	Aug. 6	-03	-04	+03
11	-03	-04	-03	14	-03	-04	+02
19	-01	-02	-02	22	-04	-06	+03
27	-02	-03	-03	30	-04	-06	+03
Mar. 7	-01	-02	-02	Sept. 7	-04	-06	+01
15	-01	-02	-01	15	-05	-08	+02
23	00	00	-01	23	-06	-09	+02
31	-01	-02	00	Oct. 1	-06	-09	+02
Apr. 8	+01	+02	00	9	-06	-09	+01
16	+01	+02	+02	17	-07	-10	+02
24	00	00	+01	25	-08	-12	+02
May 2	00	00	+02	Nov. 2	-07	-10	+02
10	-01	-01	+01	10	-06	-09	+01
18	00	00	+03	18	-09	-13	+01
26	-01	-01	+01	26	-08	-11	-01
June 3	-01	-01	+02	Dec. 4	-06	-08	-01
11	+01	+01	+02	12	-06	-08	-01
19	00	00	+01	20	-06	-08	-03
27	-02	-03	+02	28	-06	-08	-02

MERCURY, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
Jan. 2	-03	-04	-04	July 5	-06	-09	+05
10	+01	+01	-03	13	-05	-07	+05
18	+02	+03	00	21	-07	-10	+06
26	+03	+04	-01	29	-12	-18	+07
Feb. 3	+03	+04	00	Aug. 6	-12	-18	+11
11	+03	+04	+02	14	-11	-16	+04
19	00	00	00	22	-05	-07	+03
27	-02	-03	-04	30	-05	-07	-01
Mar. 7	-04	-06	-05	Sept. 7	-07	-10	+01
15	-01	-02	-05	15	-04	-06	+01
23	00	00	-04	23	-05	-08	00
31	-03	-04	-03	Oct. 1	-01	-02	00
Apr. 8	-04	-06	-02	9	00	00	-01
16	-05	-09	-01	17	00	00	-01
24	-06	-09	00	25	00	00	-01
May 2	-05	-08	+03	Nov. 2	00	00	-03
10	-02	-03	+02	10	-01	-01	-03
18	-03	-04	+01	18	-01	-01	-05
26	-06	-09	-02	26	+03	+04	-06
June 3	-09	-13	-03	Dec. 4	+08	+11	-11
11	-05	-07	-02	12	+03	+04	-03
19	-07	-10	00	20	+02	+03	-02
27	-07	-10	+04	28	-01	-01	-01

NOTE.—*Mercury* is in inferior conjunction with the Sun, 1906 November 29^d 18^h.

VENUS, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
Jan. 2	-01	-01	-12	Apr. 8	-16	-24	+01
10	+01	+01	-11	16	-16	-23	+03
18	+01	+01	-10	24	-18	-26	+03
26	-02	-03	-09	May 2	-17	-24	+05
Feb. 3	-03	-04	-08	10	-17	-24	+05
11	-05	-07	-09	18	-14	-19	+07
19	-05	-07	-06	26	-15	-21	+08
27	-08	-12	-06	June 3	-14	-19	+06
Mar. 7	-10	-15	-03	11	-12	-16	+05
15	-11	-17	-04	19	-11	-15	+05
23	-13	-20	-03	27	-08	-11	+03
31	-15	-22	-02	July 5	-09	-13	-01

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
July 13	-07	-1°0	-0°3	Oct. 9	+10	+1°4	-0°6
21	-07	-1°0	-0°6	17	+17	+2°3	-0°2
29	-07	-1°0	-0°8	25	+27	+3°6	+0°7
Aug. 6	-07	-1°1	-1°3	Nov. 2	+32	+4°2	+1°2
14	-06	-0°9	-1°3	10	+48	+6°4	+2°4
22	-06	-0°9	-1°6	18	+59	+8°0	+2°7
30	-05	-0°7	-1°5	26	+69	+9°4	+2°8
Sept. 7	-03	-0°4	-1°7	Dec. 4	+72	+10°0	+2°8
15	-01	-0°1	-1°6	12	+62	+8°8	+2°6
23	-01	-0°1	-1°3	20	+43	+6°2	+2°4
Oct. 1	+04	+0°6	-1°0	28	+29	+4°2	+2°3

NOTE.—*Venus* is in inferior conjunction with the Sun, 1906 November 29^d 17^h.

MARS, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
Jan. 2	-03	-0°4	-0°3	July 5	-11	-1°5	+0°6
10	-04	-0°6	-0°4	13	-10	-1°4	+0°6
18	-03	-0°5	-0°2	21	-11	-1°5	+0°7
26	-04	-0°6	-0°3	29	-10	-1°4	+0°7
Feb. 3	-04	-0°6	-0°3	Aug. 6	-09	-1°3	+0°9
11	-05	-0°8	-0°2	14	-09	-1°3	+0°8
19	-04	-0°6	-0°3	22	-09	-1°3	+0°8
27	-06	-0°9	-0°3	30	-08	-1°2	+0°7
Mar. 7	-06	-0°9	-0°3	Sept. 7	-08	-1°2	+0°6
15	-07	-1°0	-0°2	15	-06	-0°9	+0°7
23	-07	-1°0	-0°1	23	-07	-1°0	+0°7
31	-08	-1°2	-0°2	Oct. 1	-07	-1°0	+0°7
Apr. 8	-09	-1°3	-0°1	9	-07	-1°0	+0°4
16	-07	-1°0	+0°1	17	-08	-1°2	+0°6
24	-08	-1°1	0°0	25	-07	-1°1	+0°5
May 2	-09	-1°3	+0°1	Nov. 2	-06	-0°9	+0°4
10	-10	-1°4	+0°2	10	-06	-0°9	+0°5
18	-09	-1°2	+0°2	18	-07	-1°1	+0°5
26	-10	-1°4	+0°3	26	-06	-0°9	+0°3
June 3	-10	-1°4	+0°3	Dec. 4	-06	-0°9	+0°3
11	-09	-1°2	+0°4	12	-05	-0°7	+0°4
19	-10	-1°4	+0°6	20	-04	-0°6	+0°1
27	-11	-1°5	+0°6	28	-06	-0°9	+0°2

JUPITER, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	Time. s	R.A. Arc.	Decl. "	Day. 1906.	Time. s	R.A. Arc.	Decl. "
Jan. 2	+ '10	+ 1'4	+ 0'5	July 5	'00	0'0	+ 0'3
10	+ '09	+ 1'3	+ 0'4	13	+ '01	+ 0'1	+ 0'3
18	+ '09	+ 1'3	+ 0'5	21	- '01	- 0'1	+ 0'3
26	+ '07	+ 1'0	+ 0'2	29	- '03	- 0'4	+ 0'5
Feb. 3	+ '07	+ 1'0	+ 0'3	Aug. 6	- '01	- 0'1	+ 0'3
11	+ '08	+ 1'1	+ 0'3	14	- '02	- 0'3	+ 0'3
19	+ '06	+ 0'9	+ 0'4	22	- '03	- 0'4	+ 0'1
27	+ '09	+ 1'3	+ 0'3	30	'00	0'0	+ 0'2
Mar. 7	+ '08	+ 1'1	+ 0'4	Sept. 7	- '04	- 0'6	+ 0'2
15	+ '05	+ 0'7	+ 0'4	15	- '02	- 0'3	+ 0'3
23	+ '06	+ 0'9	+ 0'3	23	- '02	- 0'3	+ 0'2
31	+ '04	+ 0'6	+ 0'2	Oct. 1	'00	0'0	+ 0'2
Apr. 8	+ '05	+ 0'7	+ 0'4	9	- '01	- 0'1	+ 0'3
16	+ '04	+ 0'6	+ 0'3	17	- '02	- 0'3	+ 0'3
24	+ '03	+ 0'4	+ 0'3	25	- '02	- 0'3	+ 0'4
May 2	+ '02	+ 0'3	+ 0'3	Nov. 2	- '02	- 0'3	+ 0'4
10	+ '02	+ 0'3	+ 0'4	10	- '03	- 0'4	+ 0'6
18	+ '01	+ 0'1	+ 0'3	18	- '02	- 0'3	+ 0'5
26	+ '02	+ 0'3	+ 0'3	26	- '01	- 0'1	+ 0'4
June 3	'00	0'0	+ 0'3	Dec. 4	- '03	- 0'4	+ 0'7
11	+ '01	+ 0'1	+ 0'2	12	- '03	- 0'4	+ 0'4
19	+ '01	+ 0'1	+ 0'2	20	- '03	- 0'4	+ 0'6
27	+ '01	+ 0'1	+ 0'5	28	- '04	- 0'6	+ 0'5

SATURN, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	Time. s	R.A. Arc.	Decl. "	Day. 1906.	Time. s	R.A. Arc.	Decl. "
Jan. 2	- '42	- 6'1	- 2'3	Mar. 31	- '48	- 7'1	- 2'8
10	- '42	- 6'2	- 2'1	Apr. 8	- '48	- 7'1	- 2'9
18	- '42	- 6'2	- 2'3	16	- '49	- 7'3	- 3'0
26	- '42	- 6'2	- 2'4	24	- '52	- 7'7	- 3'0
Feb. 3	- '43	- 6'3	- 2'4	May 2	- '52	- 7'7	- 3'1
11	- '44	- 6'5	- 2'3	10	- '53	- 7'9	- 3'1
19	- '44	- 6'5	- 2'5	18	- '55	- 8'2	- 3'3
27	- '46	- 6'8	- 2'5	26	- '55	- 8'2	- 3'3
Mar. 7	- '47	- 6'9	- 2'7	June 3	- '57	- 8'5	- 3'5
15	- '46	- 6'8	- 2'7	11	- '58	- 8'6	- 3'5
23	- '48	- 7'1	- 2'8	19	- '60	- 8'9	- 3'6

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
June 27	-62	-9'2	-3'8	Oct. 1	-68	-10'0	-4'0
July 5	-62	-9'2	-3'9	9	-68	-10'0	-4'1
13	-63	-9'4	-4'0	17	-68	-10'0	-4'0
21	-66	-9'8	-4'0	25	-67	-9'9	-4'0
29	-65	-9'7	-3'9	Nov. 2	-67	-9'9	-3'9
Aug. 6	-67	-10'0	-4'1	10	-66	-9'7	-3'9
14	-67	-10'0	-4'0	18	-65	-9'6	-3'9
22	-70	-10'4	-4'1	26	-66	-9'7	-3'9
30	-68	-10'1	-4'0	Dec. 4	-65	-9'7	-3'9
Sept. 7	-68	-10'1	-3'9	12	-65	-9'6	-3'8
15	-68	-10'1	-3'9	20	-64	-9'5	-3'9
23	-68	-10'1	-4'0	28	-65	-9'6	-3'9

URANUS, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
Jan. 1	-31	-4'3	-0'3	July 4	-35	-4'8	-0'1
Feb. 2	-32	-4'4	-0'3	Aug. 1	-33	-4'6	-0'1
Mar. 2	-33	-4'6	-0'2	Sept. 2	-31	-4'3	-0'2
Apr. 3	-34	-4'7	-0'2	Oct. 4	-31	-4'3	-0'1
May 1	-34	-4'7	-0'1	Nov. 1	-29	-4'0	-0'1
June 2	-35	-4'8	-0'2	Dec. 3	-30	-4'1	-0'2

NEPTUNE, 1906.—*Corrections to Le Verrier's Tables.*

Day. 1906.	Time. s	R.A. Arc.	Decl.	Day. 1906.	Time. s	R.A. Arc.	Decl.
Jan. 1	-45	-6'3	-1'9	July 4	-43	-6'0	-1'6
Feb. 2	-45	-6'3	-1'9	Aug. 1	-44	-6'1	-1'7
Mar. 2	-44	-6'1	-1'9	Sept. 2	-44	-6'1	-1'7
Apr. 3	-44	-6'1	-2'1	Oct. 4	-45	-6'3	-1'8
May 1	-45	-6'3	-2'0	Nov. 1	-46	-6'4	-1'6
June 2	-44	-6'1	-1'9	Dec. 3	-48	-6'7	-1'6

H.M. Nautical Almanac Office:
1904 February 29.

Note on the Drawings of the Mare Serenitatis by John Russell, R.A. By S. A. Saunder, M.A.

On pp. 156-159 of the present volume of the *Monthly Notices* Dr. Rambaut gives an account of two drawings of the region around Linné made by Mr. John Russell, R.A., and arrives at the conclusion that they render it probable that Linné then presented much the same appearance as at the present day when viewed with a low-power eyepiece attached to a small telescope.

The drawing reproduced in plate 3, fig. 2, has not much direct bearing on the question of change, for we are told by Mädler that near the time of full moon Linné was seen in his time as here represented, namely, as a round white spot almost as white in the middle as at the edges. It is to the drawing in fig. 1 that the real interest attaches. My own introduction to this drawing was through a photograph shown me by Dr. Rambaut, and from an inspection of this I came to the conclusion that it did offer distinct evidence of a change. As I was afterwards led to modify this opinion when, by the kindness of Dr. Rambaut, I was allowed to inspect the original, it may be not without interest that I should put on record both my first impression and the reasons which led me to modify it, as many will see the reproduction and but few can hope to have the opportunity of comparing it with the original.

Referring to plate 3, fig. 1, the only genuine marks on the floor besides Bessel and the bright streaks are :

- (1) A small dark spot 0.45 in. from Bessel at position-angle 311° . This is perhaps a little more conspicuous in the original, but not much so. It almost certainly denotes Bessel A.
- (2) A somewhat larger dark mark 0.61 in. from Bessel at position-angle 352° . This is much more conspicuous in the original, but careful inspection shows that it is due to a stain and is not part of Russell's drawing.
- (3) The large dark mark 0.85 in. from Bessel at position-angle 70° . This is referred to by Dr. Rambaut in his note on p. 157, and, like the last, is a stain on the paper.
- (4) Linné, 1.37 in. from Bessel, at position-angle 53° . This, as stated by Dr. Rambaut, is whiter and more conspicuous in the original, where also the dark curved shading to the west of the white spot is clearly seen and has every appearance of being intentional. The large dark mark trending to the south-east of the white spot has its importance exaggerated in the reproduction, where it looks decidedly like a shadow, and I was led to hope that some estimate might be formed of the height of the mountain to which it was due ; but from its faintness in the original, coupled with its southerly direc-

tion, I now agree with the opinion expressed by Dr. Rambaut, that it was not intended to represent a shadow. At the base of this, however, bordering the white spot on the east side, and extending a little to the south of it, is a much darker line in the original, perhaps half as thick again as the line on the west side. It is more conspicuous in the original than in the reproduction, and conveys to my mind the impression that it was really intended to represent a shadow.

We do not know the size of the refractor most frequently employed by Russell, and can only judge of its powers and of the definition on the occasion on which the drawing was made from the drawing itself. We note that Bessel A, a conspicuous crater under this illumination in a 3 or 4 inch telescope, and one which was clearly seen by Schröter, is only just indicated. The smaller craters north and north-west of Linné are not shown at all, yet at the present time these are very clearly seen with their interiors filled with shadow under similar illumination, when in my 7-inch refractor there is no trace of shadow in the white spot surrounding Linné. It should be noted, too, that at present, when Bessel is still full of shadow, the white spot round Linné has barely developed and is far from being a conspicuous object.

It must be remembered that the special purpose of the drawing was to show the real nature of Bessel, but the great care with which the detail south of the Mare is drawn shows that other parts were not neglected. We may therefore infer that the shadow in Bessel A was only just seen, that the other small craters were not seen, and yet (if my interpretation of these two dark lines is correct) that decided shadow was seen on both sides of the bright spot, marking the position of Linné, and which, if we accept the accounts of Lohrmann, Mädler, and Schmidt, we may take as denoting the floor of the crater.

If this is admitted the drawing does furnish clear evidence of a change. But, on the other hand, it was pointed out to me by Dr. Rambaut that in the drawing representing Linné under what must have been nearly vertical illumination there are similar dark borders, nearly as thick as those under consideration, to the north and south of the white spot by which Linné is there represented. These may be seen in plate 3, fig. 2; they could not possibly have been shadows, and their existence there renders doubtful the conclusion that similar marks in fig. 1 are to be taken as real shadows.

On the whole, then, the drawing, whilst, in my opinion, quite consistent with the theory of change, cannot fairly be held to prove it, and I can only regret that after looking through these drawings I have come to the conclusion that in spite of the care with which they were made there will be very great difficulty in obtaining satisfactory information from them with regard to such small changes as there is reason to think may have occurred

from time to time upon the Moon. I may further illustrate the difficulty from the drawing now published in fig. 2. Much of the detail is evidently drawn with the greatest care, and yet the large ring shown to the north of *Cassini* does not exist upon the Moon. Careful study will show how it was suggested ; but at the same time it will also show that it was only suggested, and that a part of what is there drawn has no real existence.

I cannot conclude this short note without endeavouring to express a sense of the debt which all who are interested in this question must feel that they owe to Dr. Rambaut and the staff of the Radcliffe Observatory for the care with which they have gone into the matter and the trouble they have taken, including as it did the learning of the system of shorthand in which Russell made the only notes that we have in explanation of the drawings. That the result has not been more definite is due to the inherent difficulty of the inquiry.

On the Relative Efficiency of Different Methods of Determining Longitudes on Jupiter. By A. Stanley Williams.

1. Two chief methods have been employed for the purpose of determining the longitudes or positions of markings on *Jupiter*, and therefrom the rotation period of the planet. One of these consists in recording the times when the markings appear to be exactly in mid-transit across the disc of the planet. This is termed *the method of transits*. In the other method the positions of the markings are measured with a micrometer from the preceding and following limbs of the planet, and the time of transit derived from these measures. This is termed *the micrometric method*.

2. As is well known, very great claims have been made as to the superiority of the micrometric method over the method of transits as regards the accuracy of the results obtained. Thus, the former has been described as being "infinitely preferable" to the latter ; and positions determined by the method of transits have even been said to bear the same relation to those derived from the micrometric method, as eye-estimates of the position angles and distance of double stars possess with respect to micrometer measures of the same. As the number of positions already determined by the method of transits can hardly amount to less than 40,000, and as these constitute the foundation of much of our knowledge of the rotation of *Jupiter* and of the various surface currents known to exist, it seems desirable to make some inquiry into the relative efficiency of the two methods.

3. Reference should be made in this connection to some statements and comparisons on this subject by Professor G. W. Hough in *Astronomy and Astro-physics*, vol. xi. p. 193, and in *Popular*

Astronomy, 1903, p. 297. A few words by Professor E. E. Barnard will also be found in *Publications of the Astronomical Society of the Pacific*, vol. i. 91, and in the *Monthly Notices*, vol. lii. 11. The writer may likewise, perhaps, be permitted to refer here to a few remarks on the subject by himself published in *Popular Astronomy*, 1903, p. 188. But, somewhat singularly, in view of the strong claims that have been made as to the superiority of the micrometric method, no adequate investigation or comparison as to the relative accuracy of the results obtained by the two methods seems to have been hitherto undertaken.

4. In the "Report of the Director of the Dearborn Observatory" for 1882 it is stated that the mean error on the concluded time of transit from a single pair of measures was ± 0.9 minute, and for the "average mean probable error" for any day was ± 0.4 minute. But these values were based on observations of the red spot, when at its greatest plainness, made on thirty-one nights only; and they seem to have been derived from the accordance of the separate measures of each night *inter se*. Observers of double stars are well aware that the separate measures of a star made on the same night may be very accordant, although the mean results of several different nights may differ considerably, and for this reason it is usually considered preferable to make a small number of measures of a star on each of several different nights rather than a large number on a single night. But there are other reasons for expecting a similar but much larger difference in the case of measures of planetary markings. Nevertheless, in whatever manner derived, it is from the comparison of the above small mean or probable errors with those derived by comparing positions obtained by the method of transits on *different* nights that the supposed superiority of the micrometric method would seem to have been founded.*

5. In order to make any proper comparison between the two methods it is obviously essential to choose some mode that shall be perfectly fair and do equal justice to both. Now since an observer can only observe a transit of a spot once in the course of the same passage of the marking across the planet's disc, it is clear that it is impossible to directly derive the mean or probable error of such transit on any particular night from the observations of that night. Hence the only simple mode of comparison that appears to be suitable and yet perfectly fair to *both* methods is to compare all the observations of a spot with an ephemeris representing its motion, and thence to deduce the mean or probable error of a night's work for that particular spot. The mean result derived in this manner from a number of spots will then give a good idea of the real degree of accuracy attained by either method.

* The actual mean error of a night's work on the red spot, according to the eighty-two observations published in the above cited report, is ± 1.5 , or nearly four times greater than the figure stated above for the average mean probable error.

6. It is obviously essential not to rest our conclusions on the results obtained from one or two spots only and a limited number of observations, but to discuss a considerable amount of work accomplished by both methods in order to satisfactorily compare the two. The present paper therefore includes in the first place a discussion of the micrometrical work published by Hough in the *Monthly Notices*, vol. lx. p. 546 *et seq.* This should give a very fair idea of the general accuracy of the micrometrical method, as a good many spots were observed, and the observations of each spot are usually fairly numerous. Moreover, the very clear manner in which the work is presented renders the discussion of the results easy and pleasant. In order to render the comparison more complete, and to avoid doing any apparent injustice to the micrometric method, the fine series of observations of the red spot, eighty-two in number, made in the years 1879-82 has been included in the discussion.

7. We have in the first place, besides the results of 1879-82, four years' observations of the red spot. The first column below gives the year or years in which the observations were made, the second column the *mean error** of an observation or night's work, and the third the number of observations from which this has been derived. Then follow similar particulars for the other spots observed, in the order in which they come in the *Monthly Notices*, with an additional column descriptive of the spots. The work on the red spot has been considered separately from that done on the other spots.

The Red Spot.

Year.	Mean Error. M	Observations.
1879-82	1.5	82
1895-6	1.7	23
1897	2.0	15
1897-8	1.7	23
1898-9	1.6	27
Average mean error for red spot = $\pm$ 1.7		170

Other Spots.

Black spot B ₃ ...	...	...	1895-6	1.6	7
"	...	...	1897	2.2	9
"	...	...	1898	0.8	7
Black spot a	...	...	1895-6	2.2	16
Black spot b	...	...	1895-6	3.6	22
Black spot a	...	...	1898	2.7	18
Black spot b	...	...	1898	2.1	16

* Airy's "Theory of Errors of Observations," articles 26 and 61.

	Year.	Mean Error.	Observations.
		^M	
Black spot	1897	0.8	4
Black spot	1897	0.6	4
Black spot	1897	1.4	4
Black spot	1897	3.0	6
Black spot C	1898	1.5	7
Long black spot	1899	3.9	5
White spot	1899	1.9	14
White spot	1899	2.7	7
Average mean error for other spots = ± 2.1			146

8. It should be mentioned that three spots have not been included in the discussion, owing to their having only been observed on three nights each, and this number is too small to enable the mean error to be derived with any certainty, besides which there is necessarily usually some doubt as to identification, where there are only three observations of a spot. The first white equatorial spot referred to on p. 555 of the *Monthly Notices* has also been excluded, as the identification does not seem to the writer to be correct, the four observations given appearing to relate to three different spots. Moreover, the equatorial spots are seldom suitable for our present purpose on account of the great and rapid changes of position and appearance to which they are subject.

9. It will be seen from the foregoing table that the average mean error of an observation or night's work is $\pm 1^{\text{M}}.7$ in the concluded time of transit of the red spot, and $\pm 2^{\text{M}}.1$ in the case of other spots. With regard to the latter there are considerable variations for different spots, as might have been anticipated, for spots presenting the size and uniformity of shape and aspect of the red spot are rare. These figures differ very considerably from the $\pm 0^{\text{M}}.4$ given by Hough as the probable error of a night's work. But they are the only ones at all comparable with what has been hitherto published regarding the errors of the method of transits. Even when the red spot was at its maximum plainness (1879-82) the mean error of an observation by the micrometric method was only reduced to $\pm 1^{\text{M}}.5$; a figure which, rather curiously, is exactly the same as that derived from ninety-seven contemporary observations of Schmidt made by the method of transits, as will be seen later on. It may therefore be concluded that the times of transit of spots on *Jupiter* derived by the micrometric method are subject to an average mean error of $\pm 1^{\text{M}}.7$ in the case of the red spot and $\pm 2^{\text{M}}.1$ for other spots.

10. In order to fairly compare the results obtained by the two methods it will be necessary to determine the mean error of an observation, or night's work, in the same manner as has now been done for the micrometric method, for a number of spots observed by the method of transits. Only comparatively few of

the numerous observations made according to this method have been published in the very clear and detailed manner that has been employed by Hough, so that in the great majority of cases a considerable amount of labour and time would necessarily have to be devoted to the purpose in order to extract the necessary information. Nevertheless sufficient material is already to hand to enable the mean error to which the method of transits is subject to be determined in a manner comparable with that done for the micrometric method.

11. The results contained in the following table have been extracted from various sources, and are based on the work of several observers. The list is not by any means supposed to be exhaustive, but, as every instance found by the writer and published in sufficient detail has been made use of, it is probable that they are fairly representative of the method. No attempt has been made to select cases in which the observations are specially accordant or to reject any that are unusually discordant. The only cases excluded are two or three instances of spots of which only three or four observations are available, and where consequently a satisfactory value of the mean error cannot be determined. The table contains the same particulars as the previous one, with the addition in the last column of the name of the observer and a reference to the source from which the result has been derived.

The Red Spot.

Year.	Mean Error.	Observations.	Observer.
1880-2	$1^{\circ}0\frac{1}{2}$	44	Barnard (<i>Pub. A.S.P.</i> , i. 91)
1891	$0^{\circ}8^*$	11	„ (<i>Monthly Notices</i> , lii. 12)
1898	$2^{\circ}2\frac{1}{2}$	10	Denning (<i>ibid.</i> , lix. 82)
1899	$1^{\circ}3\frac{1}{2}$	35	„ (<i>ibid.</i> , lx. 217)
1898	$1^{\circ}5\frac{1}{2}$	14	Gledhill (<i>ibid.</i> , lix. 82)
1899	$1^{\circ}0\frac{1}{2}$	23	„ (<i>ibid.</i> , lx. 217)
1880-1	$0^{\circ}5$	11	Jedrzejewicz (<i>A.N.</i> , 2366)
1898	$1^{\circ}4\frac{1}{2}$	8	Phillips (<i>Monthly Notices</i> , lix. 82)
1899	$1^{\circ}6\frac{1}{2}$	26	„ (<i>ibid.</i> , lx. 217)

† Barnard states that “among the observations of the red spot I have forty-four complete and carefully estimated transits—that is, observations of the preceding end, middle, and following end of the spot. Twenty-one of these are from a single but careful estimate of each phase. These give the probable error of a transit of the centre from the mean of the three observations = $\pm 1^{\text{m}}0$. In twenty-three of these transits three estimations were made of each phase: from these I get for the transit of the middle from the mean of the nine observations the error of the transit, = $\pm 0^{\text{m}}.7$ ” (*Pub. A.S.P.* vol. i. p. 91). The mean error of a transit for 1880-82, the period in which these 44 observations were made, has been assumed to be $\pm 1^{\text{m}}0$ in the table. In all other cases the mean error has been derived directly by the writer.

† In these cases the “hollow” or bay in the S. equatorial belt was observed, and not the actual spot. Owing to the faintness of the latter all observers except the writer seem during the last few years to have relinquished observing the spot in favour of the “hollow.”

Year.	Mean Error.	Observations.	Observer.
	^M		
1879-81	1·5	97	Schmidt (<i>A.N.</i> , 2410)
1892	2·0*	25	Williams (<i>Mem. B.A.A.</i> , ii. 143)
1893-4	2·6*	14	„ (<i>ibid.</i> , iii. 138)
1898	2·0	15	„ (<i>Monthly Notices</i> , lix. 82)
1899	3·1	9	„ (<i>A.N.</i> , 3596)
1900	0·8	4	„ (<i>ibid.</i> , 3675)
1901	2·3	14	„ (<i>ibid.</i> , 3786)
1902	2·2	22	„ (<i>ibid.</i> , 3875)
Average mean error = ± 1·6		382	

Other Spots.

	Year.	Mean Error.	Observations.	Observer.
		^M		
Dark spot ...	1895-6	2·4	13	Brenner (<i>Monthly Notices</i> , lvi. 531)
White spot I...	1898	2·9	6	Denning (<i>ibid.</i> , lix. 85)
Dark ellipse ...	1898	2·7	5	„ (<i>ibid.</i> , lix. 87)
Dark spot II...	1898	(7·5)	6	„ (<i>ibid.</i> , lix. 88)
Dark spot ...	1895-6	2·0	30	Gledhill (<i>ibid.</i> , lvi. 531)
Dark spot ...	1895-6	2·4	5	Merfield (<i>ibid.</i> , lvi. 531)
Dark ellipse ...	1898	(5·8)	8	Phillips (<i>ibid.</i> , lix. 87)
Dark spot ...	1898	2·1	9	„ (<i>ibid.</i> , lix. 88)
Dark spot ...	1895-6	0·7†	7	Rambaut (<i>ibid.</i> , lvi. 531)
Dark spot ...	1862	0·6	12	Schmidt (<i>A.N.</i> , 2342)
Sharp elbow (Ecke)	1862	3·0	6	„ (<i>ibid.</i> , 1973)
White spot ...	1866	1·2	5	„ (<i>ibid.</i> , 1973)
White spot ...	1890	2·6*	20	Williams (<i>Monthly Notices</i> , lviii. 323)
End of belt ...	1881	1·2	9	„ (<i>ibid.</i> , lix. 29)
Dark spot ...	1888	2·6	10	„ (<i>ibid.</i> , lix. 30)
White spot II.	1898	1·0	5	„ (<i>ibid.</i> , lix. 84)
White spot I...	1898	2·7	5	„ (<i>ibid.</i> , lix. 85)
White spot III.	1898	0·6	5	„ (<i>ibid.</i> , lix. 85)
Dark spot ...	1892	1·7	16	„ (<i>A.N.</i> , 3528)
Dark spot ...	1890	2·5	8	„ (<i>ibid.</i> , 3528)
Average mean error = ± 1·9		176		

12. There are some circumstances affecting the results contained in the foregoing table that do not favour the method of

† It is assumed that Rambaut's observations were made by the method of transits, but the writer has not seen the original paper from which they were taken.

transits. In the cases marked with an asterisk the motion of the spot was assumed to conform to Marth's ephemeris. This, however, was probably never exactly so, and consequently the stated values of the mean error are no doubt somewhat too great. Also in several cases the observations of a spot were compared with an ephemeris representing its mean motion as derived from the work of several observers, and a discussion based solely on the observations of the particular observer would probably result in reducing the residuals, and so the mean error. It may therefore be safely assumed that the average mean errors derived from the table are, if anything, too large. Schmidt's fine observations of the red spot are corrected for his "Correction C," representing a constant error varying according to the hour-angle of *Jupiter* at the time of observation. Schmidt seems to have connected this constant error with the inclination of the belts to the line joining the two eyes when the planet is far east or west of the meridian, and the head is held level. Probably he was right in this, as no similar error seems to be revealed by the work of other observers, most of whom, it is believed, take the precaution of keeping the eyes parallel to the belts when observing the transits. The two bracketed results in the list of "other spots" have been omitted in forming the average, as they are largely in excess of any others, and relate to two faint and difficult spots in a rather high S. latitude.

13. We are now in a position to make a proper comparison of the results obtained by the two methods. We have:—

<i>Red Spot.</i>			
	Average Mean Error.	No. of Series or Spots.	No. of Obs.
Micrometric Method ...	± 1.7	5	170
Method of Transits ...	± 1.6	17	382
<i>Other Spots.</i>			
Micrometric Method ...	± 2.1	15	146
Method of Transits ...	± 1.9	18	176

It will be seen that, so far from the micrometric method showing any superiority in point of accuracy over the method of transits, the advantage is rather in favour of the latter. The difference is, however, only slight, and it may therefore be concluded that the two methods yield results of about equal accuracy.

14. By largely increasing the number of measures of a spot theoretically* greater accuracy could be secured by the micrometric method than by the method of transits, though it would

* Practically it seems doubtful, however, whether any real increase in accuracy would result, on account of liability to bias in the later measures. Some years ago, when measuring some photographs of *Jupiter*, the writer found this liability to bias so marked as to render it expedient not to measure the different images of the planet in order of time; but this cannot be managed in the case of direct measures on the planet at the telescope.

seem that the beneficial effect of this would be slight. For the mean error is no doubt made up chiefly of three quantities: (a) the real errors of observation; (b) apparent errors due to the motion of the spot not being perfectly uniform, and (c) apparent errors produced by changes in the form and appearance of the spot. The apparent errors *b* and *c* will balance themselves when a large number of observations and many spots are discussed, so that their effect may be considered as the same for both methods. Now if $\pm 0^{\text{M}}.4$ be taken to represent the real error, then doubling the accuracy of the observations would only result in reducing the mean error from, say, $2^{\text{M}}.0$ to $1^{\text{M}}.8$. Considering the nature of the problems requiring to be investigated, it seems doubtful whether the great increase in the labour and time necessitated by the larger number of measures would be compensated for by the slight benefit obtained.

15. It has been said that an eye-estimate of a transit is, in a manner, equivalent to a single micrometer measure, and liable to the same error of bisection, &c.; but this, I think, is clearly not so. In a complete single measure, as described by Hough, one micrometer wire is placed tangent to the preceding limb of the planet, and the other made to bisect the spot. Both wires are then carried across the disc, and one placed tangent to the following limb and the other made to bisect the spot. It would seem, therefore, that in every complete measure there are four distinct sources of error. There is the liability to error in twice placing the wires tangent to the two limbs, and there is the liability to error in twice bisecting the spot. In the method of transits the comparisons are made direct with the true limbs, so that the liability to error in placing the wires tangent to the latter does not arise. Then the spot itself takes the place of the other wire, and there is consequently only a single bisection here—the bisection of the disc. Hence there are four distinct sources of error to which a single measure by the micrometric method is liable, compared with one in the method of transits. The effect produced by the rapid rotational motion of the spots must also not be lost sight of in making comparisons of this kind. If the rotation of *Jupiter* were performed in one hour, instead of nearly ten, the advantage would be all on the side of the method of transits.

16. It is not proposed to consider here the question of personal equation. It is only of importance where contemporary observations of the same marking by two or more observers are combined, but there is no reason for supposing that it would be any less in the case of the micrometrical method than it is known to be where the method of transits is used. It is no doubt chiefly due to different observers not observing markings, irregular both in shape and intensity, and often very highly so, in exactly the same way.

17. The conclusions derived from the foregoing investigation may be shortly stated as follows:

- (1) The alleged great or indeed any superiority in accuracy of the micrometric method over the method of transits does not exist.
- (2) The two methods give results of about equal accuracy.
- (3) The times of transit derived from either method are subject to an average mean error of $\pm 2^{\text{M}}.0$, but in the case of very prominent and definite spots, such as the red spot in the years 1879-82, the mean error may be reduced to $\pm 1^{\text{M}}.5$ or even less. On the other hand, it often largely exceeds $2^{\text{M}}.0$, especially in the case of faint or irregular markings.

18. As the rotation of *Saturn* is performed in nearly the same time as that of *Jupiter*, it is evident that the first two conclusions apply with equal force to the former planet ; so that the statement made in the *Monthly Notices*, vol. lxiv. 122, as to the great superiority of the micrometric method for determining the positions of spots on *Saturn* does not appear to be justified.

Hove : 1904 February 12.

Positions and Photographic Magnitudes of ninety Stars surrounding the Variable R Cygni. By Joel H. Metcalf, Ph.D.

(Communicated by Professor H. H. Turner.)

1. The suggestion that such work as the following might have value came from the recent experience of the Oxford Observatory in the work on the Rousdon variable stars. Existing positions for the comparison stars are usually given only roughly, and sometimes there is a doubt about identification in consequence. It was therefore felt that accurate places would be desirable. In this particular case it will be seen that the places good enough for a working chart at Rousdon were sometimes $5'$ in error.

2. There would also seem to be good general reasons for accurate surveys of special regions, especially those that were previously known to contain objects of interest. On this account the region of variables is sure to be under close observation, and a detailed study of them will be given which it will not be possible to extend to the whole sky.

3. The photographic plate, which is the basis of the following catalogue, was taken at the University Observatory with the 13-inch astrographic telescope on the night of 1903 November 15. The plate was exposed from $1^{\text{h}} 8^{\text{m}} 50^{\text{s}}.0$ to $2^{\text{h}} 9^{\text{m}}$ local sidereal time. As a few seconds of time were lost in the middle of the exposure the time was approximately one hour. The plate is a good one and shows stars of about the 14th magnitude, as one

would judge by comparison with stars of known brightness. The variable itself has a diameter of 614, or 18''·42, which from the formulæ which were used would give a magnitude of M_1 7·4 and M_2 7·8 respectively.

The centre of the plate adopted was R.A. 19^h 32^m 56^s, D. +49° 52' 30''·0, which is approximately the position of the variable. It was measured in the following weeks by myself in the direct and reverse positions, as is the custom in the work on the astrographic chart, with the ordinary micrometer of the observatory, with the réseau and divided scale in the eye-piece.

4. To obtain the constants of the plate twelve stars were selected from the A.G. Catalogues of Bonn and Harvard, which were distributed in four groups of three each as near as possible to the edge of the plate. To Bonn 13117 the proper motion given in the catalogue was applied. To the others none was applied.

For these twelve stars standard coordinates were calculated in units of 5' = 1 réseau interval for epoch 1855, referred to plate centre 19^h 32^m 56^s·0 + 49° 52' 30''·0.

TABLE I.

Star.	Standard Coords.		Δx_1 .	Δy_1 .	Δx_2 .	Δy_2 .
	$x+13^{\circ}0000$.	$y+13^{\circ}0000$.				
East						
B 13316	24·1407	10·1577	—·3867	+1·1613	+·0016	—·0030
B 13323	24·4883	14·8213	—·4503	+1·1632	+·0009	—·0005
H 6152	22·9616	18·3597	—·4966	+1·1448	+·0008	+·0022
West						
B 13102	2·4846	14·5730	—·4276	+·8925	+·0019	+·0005
B 13101	2·4267	16·0430	—·4447	+·8870	+·0046	—·0030
B 13117	3·8283	12·9422	—·4118	+·9103	—·0027	+·0003
North						
H 6095	9·9691	25·2356	—·5826	+·9779	—·0037	+·0025
H 6116	15·8947	24·1169	—·5687	+1·0466	·0000	—·0033
H 6129	18·2566	24·1675	—·5716	+1·0820	—·0004	+·0026
South						
B 13230	14·9414	1·9205	—·2669	+1·0655	+·0035	+·0078
B 13228	14·6251	2·5349	—·2831	+1·0476	—·0048	—·0052
B 13244	17·1651	4·5080	—·3091	+1·0820	—·0023	—·0007

Under Δx_1 , Δy_1 are given the differences directly obtained from the measures. From these the formulæ for correction were found to be

$$\Delta x_1 = -\cdot00066x - \cdot01343y - \cdot02208$$

$$\Delta y_1 = +\cdot01243x - \cdot00070y + \cdot08772$$

Applying these corrections to the differences, we obtain the residuals shown as Δx_2 and Δy_2 .

5. Applying these constants of reduction to the stars measured, and afterwards reducing the standard coordinates to R.A. and Decl., we obtain the results shown in Table II.

Columns 2 and 3 refer to Father Hagen's Atlas of Variable Stars, and

Columns 4 and 5 to the *Memoirs R.A.S.* Vol. LV. p. xxxiii (now in the press). The magnitudes were formed by adding to the "Adopted Mag." the adopted correction given in the last column whenever available. The rough places there given are sometimes as much as 5' in error, as already remarked in § 1; but it is hoped that the stars have been correctly identified.

In column 6 is given the diameter of the star disc on the photograph measured in units of 0''·03 (or 0·0001 réseau intervals).

In columns 7 and 8 magnitudes are deduced from these diameters from two formulæ hereafter to be explained.

TABLE II.
Stars near R Cygni.

Ref. No.	Hagen		Rousdon		Photo- graphic Diameter.	M ₁ .	M ₂ .	Coordinates, Epoch 1855·0.			
	No.	Mag.	No.	Mag.				Standard		P.A. 19 ^h + m s	N. Decl. 49° + ' "
								X.	Y.		
1	...	...	8	9·8	347	10·5	10·3	-5·921	-0·074	29 52·29	51 58·7
2	19	9·5	...	...	469	9·1	9·0	-4·633	+4·995	30 30·97	77 22·9
3	...	...	15	11·0	170	12·5	13·4	-4·554	+1·263	30 34·36	58 43·5
4	20	9·6	...	...	398	9·9	9·7	-4·297	-2·580	30 43·25	39 31·2
5	5	8·6	3	8·8	500	8·7	8·7	-4·137	+1·240	30 47·34	58 37·6
6	24	9·8	...	...	439	9·4	9·3	-3·688	-3·225	31 2·18	36 19·0
7	4	8·4	4	8·5	644	7·0	7·6	-3·233	+5·176	31 14·77	78 20·0
8	40	11·3	...	...	295	11·1	11·0	-2·747	+0·812	31 30·63	56 31·7
9	42	11·4	...	...	297	11·1	11·0	-2·734	+1·848	31 30·87	61 42·5
10	37	10·9	...	...	349	10·5	10·3	-2·601	-0·945	31 35·41	47 44·8
11	58	12·4	...	...	246	11·7	12·2	-2·357	+1·747	31 42·63	61 12·7
12	12	9·1	...	...	424	9·6	9·4	-2·363	-5·366	31 43·34	25 38·8
13	25	9·8	...	...	484	8·9	8·9	-2·332	-1·830	31 43·86	43 19·6
14	47	11·7	...	...	236	11·8	12·0	-2·288	+1·840	31 44·77	61 40·7
15	61	12·5	...	...	140	12·9	14·2	-2·177	+2·820	31 48·11	66 34·8
16	44	11·5	...	...	295	11·1	11·0	-2·177	-1·253	31 48·59	46 12·9
17	48	11·7	...	...	220	12·0	12·3	-2·029	-0·141	31 53·06	51 46·7
18	28	10·0	...	...	407	9·8	9·6	-1·988	-2·217	31 54·54	41 23·9
19	57	12·2	...	...	220	12·0	12·3	-1·783	+2·597	32 0·42	65 28·3
20	...	...	14	11·5	204	12·1	12·6	-1·759	-0·135	32 1·44	51 48·7
21	18	9·5	...	...	408	9·8	9·7	-1·760	-3·560	32 1·71	34 41·2
22	53	11·9	...	...	181	12·4	13·1	-1·738	+2·209	32 1·86	63 31·9

Ref. No.	Hagen		Rouslon		Photo- graphic Diameter.	M ₁ .	M ₂ .	Coordinates, Epoch 1855 ^o 0.		R.A. 19 ^h + 8	N. Decl. 49 ^o +
	No.	Mag.	No.	Mag.				Standard			
								X.	Y.		
23	7	8.9	...	...	520	8.5	8.5	-1.740	-6.074	32 2.56	22 7.0
24	2	7.0	2	7.4	800	5.2	6.7	-1.711	+0.488	32 2.86	54 55.7
25	22	9.7	...	...	395	9.9	9.7	-1.530	+5.263	32 8.09	78 48.3
26	17	9.4	...	...	394	9.9	9.7	-1.465	-4.173	32 10.86	31 37.6
27	51	11.9	13	11.5	220	12.0	12.3	-1.443	-0.809	32 11.28	48 26.8
28	38	11.1	...	...	326	10.8	10.6	-1.419	+1.418	32 11.87	59 34.9
29	50	11.8	...	...	246	11.7	11.8	-1.369	-0.112	32 13.52	51 55.9
30	...	...	...	...	164	12.6	13.6	-1.287	-0.225	32 16.08	51 22.1
31	34	10.7	...	...	287	11.2	11.1	-1.248	+2.711	32 17.09	66 2.9
32	...	...	...	...	136	13.0	14.4	-1.141	-0.035	32 20.59	52 19.2
33	...	...	...	...	100	13.3	15.7	-1.126	+0.170	32 20.96	53 20.7
34	10	9.0	...	...	465	9.1	9.0	-1.096	-4.489	32 22.24	30 3.0
35	...	...	...	...	100	13.3	15.7	-1.076	-0.769	32 22.65	48 39.0
36	...	...	...	...	100	13.3	15.7	-1.043	+0.848	32 23.58	56 44.1
37	...	...	...	...	100	13.3	15.7	-0.983	+0.635	32 25.46	55 40.3
38	...	...	...	...	100	13.3	15.7	-0.858	-0.769	32 29.41	48 39.1
39	...	...	...	...	240	11.8	11.9	-0.744	+0.038	32 32.91	52 41.3
40	1	4.9	1	6.4	1440	-2.2	4.1	-0.734	+0.182	32 33.22	53 24.5
41	...	...	...	...	150	12.8	13.9	-0.724	-0.189	32 33.55	51 33.2
42	55	12.1	...	...	248	11.7	11.8	-0.590	+1.251	32 37.65	58 45.2
43	60	12.5	...	...	141	12.9	14.2	-0.584	-0.544	32 37.89	49 46.7
44	...	...	17	12.5	215	12.0	12.4	-0.560	+0.408	32 38.61	54 32.3
45	36	10.8	11	10.7	352	10.5	10.2	-0.507	-1.473	32 40.30	45 8.0
46	...	...	...	...	120	13.1	15.0	-0.398	+0.179	32 43.65	53 23.7
47	56	12.2	...	...	284	11.2	11.2	-0.296	-0.460	32 46.82	50 12.0
48	...	...	...	...	100	13.3	15.7	-0.132	-0.182	32 51.91	51 35.4
49	...	...	16	13.3	210	12.1	12.5	-0.121	-0.424	32 52.25	50 22.8
50	...	...	...	...	100	13.3	15.7	-0.090	+0.098	32 53.21	52 59.4
51	59	12.4	12	11.8	218	12.0	12.3	-0.047	-0.587	32 54.54	49 33.9
52	R Cygni	...	...	...	614	7.4	7.8	-0.016	-0.003	32 55.52	52 29.1
53	14	9.3	10	9.6	488	8.9	8.8	+0.061	+0.294	32 57.89	53 58.2
54	...	...	...	...	100	13.3	15.7	+0.366	+0.866	33 7.37	56 49.8
55	...	...	...	...	160	12.6	13.7	+0.420	+0.537	33 9.05	55 11.1
56	...	...	...	...	100	13.3	15.7	+0.466	+0.875	33 10.48	56 52.5
57	49	11.8	...	...	193	12.3	12.8	+0.472	-1.773	33 10.60	43 38.1
58	23	9.8	...	...	420	9.7	9.5	+0.501	+3.658	33 11.64	70 47.3
59	...	...	...	...	100	13.3	15.7	+0.623	-0.311	33 15.32	50 56.6
60	...	...	...	...	100	13.3	15.7	+0.636	-1.030	33 15.70	47 20.9

Ref. No.	Hagen		Rousdon		Photo- graphic Diameter.	M ₁ .	M ₂ .	Coordinates, Epoch 1855'0.				R.A. 19 ^h +	N. Decl. 49°+
	No.	Mag.	No.	Mag.				Standard		m			
								X.	Y.				
61	...	...	...	...	100	13'3	15'7	+0'688	+0'913	33	17'38	57	3'8
62	...	...	...	...	110	13'2	15'3	+0'743	+0'134	33	19'06	53	10'1
63	27	10'0	...	...	335	9'9	9'7	+0'763	+2'579	33	19'78	65	23'6
64	...	...	...	...	100	13'3	15'7	+0'838	+0'962	33	22'06	57	18'4
65	35	10'8	...	...	327	10'8	10'6	+0'870	+1'356	33	23'07	59	16'6
66	43	11'4	...	...	269	11'4	11'4	+0'909	-1'301	33	24'16	45	59'5
67	...	...	...	...	200	12'2	12'7	+0'911	-0'135	33	24'27	51	49'3
68	54	11'9	...	...	226	11'9	12'2	+0'956	-2'162	33	25'56	41	41'2
69	...	...	...	...	100	13'3	15'7	+0'981	+0'449	33	26'47	54	44'5
70	39	11'2	...	...	294	11'1	11'0	+0'985	+1'303	33	26'64	59	0'7
71	...	...	...	...	110	13'2	15'3	+1'111	-0'191	33	30'47	51	32'4
72	46	11'7	...	...	207	12'1	12'5	+1'186	-2'283	33	32'66	41	4'8
73	52	11'9	...	...	203	12'2	12'7	+1'194	-2'579	33	32'89	39	35'9
74	41	11'4	...	...	269	11'4	11'4	+1'209	+1'268	33	33'60	58	50'0
75	...	...	...	...	100	13'3	15'7	+1'253	+0'705	33	34'93	56	1'1
76	8	8'9	...	...	504	8'7	8'7	+1'288	+4'759	33	36'30	76	17'3
77	31	10'2	...	...	413	9'7	9'5	+1'420	+0'563	33	40'11	55	18'4
78	45	11'6	...	...	293	11'1	11'0	+1'489	-0'369	33	42'18	50	38'7
79	32	10'3	...	...	323	10'8	10'6	+1'610	-0'214	33	45'94	51	25'1
80	29	10'0	...	...	333	10'7	10'5	+2'001	+1'230	33	58'23	58	38'0
81	30	10'1	...	...	355	10'4	10'2	+2'098	+1'908	34	1'32	62	1'3
82	33	10'5	...	...	349	10'5	10'3	+2'473	-0'570	34	12'67	49	37'4
83	11	9'1	5	9'2	424	9'6	9'4	+2'618	-1'383	34	17'05	45	33'4
84	16	9'4	...	...	415	9'7	8'8	+3'013	+3'226	34	30'03	68	35'4
85	6	8'8	6	8'8	418	9'7	9'5	+3'468	-0'002	34	43'62	52	26'3
86	15	9'3	...	...	388	10'0	9'8	+3'440	+5'672	34	43'81	80	48'5
87	13	9'1	7	9'1	407	9'8	9'6	+4'370	+1'510	35	11'97	59	58'0
88	26	9'9	...	...	418	9'7	9'5	+4'930	+3'931	35	30'04	72	2'9
89	21	9'6	...	...	352	10'4	10'2	+5'189	+5'351	35	38'54	79	8'2
90	9	9'0	...	...	405	9'8	9'6	+5'655	-2'352	35	50'78	40	36'2
91	3	8'2	...	...	648	7'0	7'6	+8'469	+2'034	37	19'71	62	21'5

6. To convert the diameters of the star images into photographic magnitudes they were first plotted against Hagen's magnitudes, and it was found that a straight line would very fairly represent the results from mag. 12'5 to 8'5, but for the brighter stars the diameters were too large.

The formula deduced was

$$\text{Magnitude} = 14'5 - 0'116 \times \text{diameter.}$$

The magnitudes computed by using it are given under heading 7, designated M_1 . It will be seen that the Hagen stars Nos. 1 to 4 cannot be represented by this even approximately.

Accordingly the log (diameter) was plotted against the Hagen magnitudes, and it was found that the following straight line represented the results very nearly :—

$$\text{Magnitude} = 35.7 - 10 \log (\text{diameter}).$$

This formula is much better for the brighter stars, but fails for the fainter ones.

It may be noticed that on transforming it to the form

$$\text{Intensity of light} \propto 10^{-0.4 \times \text{mag.}} \propto (\text{diameter})^4$$

this formula corresponds to the supposition that the diameter increases as the fourth root of the exposure time, if we suppose exposure time exactly equivalent to increased intensity of light. This law was obtained by Pritchard, *Proc. R.S.* vol. xl., though other laws have been found by other observers—the inverse square by Rutherford and the inverse cube by Prof. Turner at Greenwich in 1889 (see *Monthly Notices*, xlix. p. 292).

The results from this formula are given under head M_2 .

7. Father Hagen gave the places of his comparison stars only approximately to the nearest second of time in R.A. and the nearest 0'.1 in D, and so no comparison is printed, though one was made and sent to him. The differences between his positions and those obtained by me are often quite large. This would seem to show the need of similar work being done for the whole list of variable stars.

Oxford University Observatory,

On the Absolute Proper Motions of Certain Double Stars showing Large Relative Motion. By H. FURNER and J. STOREY.

(Communicated by the Astronomer-Royal.)

During Mr. Lewis' work on double stars he has found a number which show large relative motions, and it is of interest to examine the meridian observations in order to determine these absolute motions. A list of these stars was given to us, and a first instalment of our work upon them we now beg to lay before the Society.

Some 100 stars have been examined, but owing to the lack of sufficient material we have confined our present paper to such stars as show by micrometer measures a relative motion of at least 0".1 per annum.

All available catalogue places have been made use of and reduced to epoch 1900.0, applying Professor Auwers' systematic corrections as given in *Ast. Nach.* 3195-6 and 3463. The stars from the Lalande Catalogue were reduced to the epoch 1800 by the help of Dr. E. von Asten's tables (*Vierteljahrschrift der Astronomische Gesellschaft*, Jahrgang III., Supplement). The graphical method has been employed in determining the proper motions, greater weight having been given to Struve's *Positiones Medice* 1830.0 for the earlier catalogues. It is of interest to note that in about half the stars here considered the larger proper motion belongs to the fainter component; a circumstance which may be explained by the fact that if the brighter star had the larger proper motion in all probability this would have already been detected.

Catalogue.	Epoch.	R.A. 1900.0.	P.M. applied.	N.P.D. 1900.0.	P.M. applied.
Σ 63. Mag. 8.2, 11.2.					
		$0^h 44^m$		$78^{\circ} 42'$	
W. B. ...	1825	58.47	58.63	41.6	45.0
Pos. Med. ...	32.4	58.62	58.76	45.4	48.4
Camb. Obs. ...	43.9, 42.8	58.68	58.79	45.7	48.3
Göttingen ...	60	58.75	58.83	42.1	43.9
Leipzig ...	71.4	58.71	58.77	48.2	49.5
Washington ...	89.7	58.72	58.74	46.4	46.9
Cincinnati ...	89.8	58.72	58.74	48.9	49.4
Greenwich ...	89.8	58.75	58.77	48.4	48.9
Cape ...	89.8	58.76	58.78	47.9	48.4
Radeliffe ...	90.9	58.74	58.76	47.6	48.0

P.M. of A $+0''.031$ $+0''.045$

The relative motion of B to A from micrometer measures :

$-0''.159$ $0''.000$

P.M. of B $-0''.128$ $+0''.045$

Σ 175. Mag. 7.9, 8.4.

		$1^h 45^m$		$69^{\circ} 22'$	
Pos. Med. ...	1835.5	30.92	31.03	44.4	47.9
Berlin B....	82.0	31.00	31.03	46.9	47.9

P.M. of A $+0''.024$ $+0''.054$

The relative motion of B to A from micrometer measures :

$-0''.026$ $-0''.098$

P.M. of B $0''.000$ $-0''.044$

Catalogue.	Epoch.	R.A. 1900'0.	P.M. applied.	N.P.D. 1900'0.	P.M. applied.
Σ 197. Mag. 7.2, 8.2.					
		^h 55 ^m		55° 10'	
Lalande ...	1794.9	10.27	10.74	59.0	50.1
W. B. ...	1828.8	10.56	.90	51.6	45.6
Pos. Med. ...	33.8	10.58	.90	54.3	48.7
Brussels ...	67.6	10.60	.75	50.8	48.1
Leiden ...	73.5	10.73	.85	50.7	48.4

P.M. of A +0''.056 -0''.085

The relative motion of B to A from micrometer measures :

-0''.082 +0''.064

Σ 436. Mag. 7.0, 8.2.

		^h 36 ^m		° 56'	
W. B. ...	1825	7.94	8.19	22.6	20.4
Pos. Med. ...	36.8	7.96	.17	24.0	22.2
Schjellerup ...	65	7.88	.00	26.1	25.1
Radcliffe Obs. ...	67.9	7.96	.07	...	...
Radcliffe... ..	91.7	8.16	.19	22.4	22.2

P.M. of A +0''.050 -0''.029

The relative motion of B to A from micrometer measures :

-0''.185 +0''.040

P.M. of B -0''.135 +0''.011

Σ 499. Mag. 9.2, 9.3, 11.2.

		^h 3 ^m		66° 11'	
Lalande ...	1798.4	44.17	44.36	22.1	...
W. B. ...	1825	44.16	.30	25.9	...
Pos. Med. ...	27.0	44.08	.22	25.7	...
Rumker ...	36	43.97	.09	24.7	...
Romberg ...	75.5	44.26	.31	25.5	...
Berlin B....	80.7	44.20	.24	26.1	...

P.M. of A +0''.029 0''.000.

The micrometer measures show that the stars A and B are relatively fixed ; hence A and B have the same P.M. The relative motion of C to A from micrometer measures :

-0''.210 -0''.037

P.M. of C -0''.181 -0''.037

Catalogue.	Epoch.	R.A. 1900'0.	P.M. applied.	N.P.D. 1900'0.	P.M. applied.
Σ 853. Mags. 7.8, 8.3.					
		$6^h 3^m$ s		$78^{\circ} 19'$	
Pos. Med. ...	1833.7	35.59	35.11	34.5	...
Schjellerup ...	65	35.31	.06	32.5	...
Leipzig I. ...	70.6	35.26	.04	35.4	...
Washington ...	76.9	35.31	.14	34.1	...

P.M. of A $-0''.106$ $0''.000$

The relative motion of B to A from micrometer measures :

$+0''.071$ $-0''.084$
P.M. of B $-0''.035$ $-0''.084$

Σ 1142. Mag. 8.0, 10.4.					
		$7^h 42^m$ s		$76^{\circ} 20'$	
Lalande ...	1796.2	46.96	46.74	3.9	2.5
W. B. ...	1825	47.23	47.07	2.7	1.7
Pos. Med. ...	37.2, 37.7	46.91	46.78	2.1	1.3
Leipzig I. ...	70.6	46.87	.81	2.4	2.0
Paris ...	74.1, 75.1	46.80	.75	1.7	1.4

P.M. of A $-0''.031$ $-0''.013$

The relative motion of B to A from micrometer measures :

$+0''.043$ $+0''.175$
P.M. of B $+0''.012$ $+0''.162$

Σ 1202. Mag. 7.7, 9.8.					
		$8^h 3^m$ s		$78^{\circ} 50'$	
Lalande ...	1796.2	5.13	4.91	54.2	60.4
Piazzi ...	1800	5.23	5.03	55.9	61.9
W. B. ...	25	5.09	4.94	53.2	57.7
Pos. Med. ...	32.7	5.05	4.92	55.9	59.9
Madras ...	36.4	5.25	5.12	55.8	59.6
Paris, 1860 ...	60.6, 60.1	4.99	4.91	57.2	59.6
Leipzig I. ...	69.5	4.98	4.92	57.7	59.5
Romberg ...	75.4	5.00	4.95	58.5	60.0
Paris, 1875 ...	79.2	5.00	4.96	59.8	61.0

P.M. of A $-0''.029$ $+0''.060$

The relative motion of B to A from micrometer measures :

$-0''.105$ $+0''.060$
P.M. of B $-0''.134$ $+0''.120$

Catalogue.	Epoch.	R.A. 1900.0.	P.M. applied.	N.P.D. 1900.0.	P.M. applied.
Σ 1329. Mag. 8.3, 8.5.					
		$9^h 10^m$		$90^\circ 49'$	
Pos. Med. ...	1827.2	$38^s 67$	$38^s 30$	$21^s 1$	$27^s 0$
Munich ...	42.9	$38^s 52$	$38^s 24$	$21^s 7$	$26^s 3$
Nicolajew ...	88.5	$38^s 33$	$38^s 27$	$26^s 2$	$27^s 1$

P.M. of A $-0''.075$ $+0''.081$

The relative motion of B to A from micrometer measures :

$+0''.065$ $-0''.099$

Σ 1847. Mag. 8.5, 9.8.					
		$14^h 23^m$		$99^\circ 45'$	
W.B. ...	1825	$18^s 09$	$17^s 90$	$22^s 5$	$29^s 3$
Camb. Obs. ...	1843.2	$18^s 40$	$18^s 26$	$22^s 0$	$27^s 1$
Pos. Med. ...	43.3, 45.5	$18^s 59$	$18^s 45$	$21^s 7$	$26^s 7$
Munich... ..	61.4	$18^s 40$	$18^s 30$	$24^s 2$	$27^s 7$
Paris	63.4, 60.7	$18^s 53$	$18^s 44$	$24^s 4$	$27^s 9$
Romberg	75.0	$18^s 50$	$18^s 44$	$24^s 5$	$26^s 7$

P.M. of A $-0''.035$ $+0''.090$

The relative motion of B to A from micrometer measures :

$-0''.105$ $-0''.042$
P.M. of B $-0''.140$ $+0''.048$

Σ 1893. Mag. 8.4, 10.0.					
		$14^h 51^m$		$60^\circ 7'$	
Lalande	1795.5	$1^s 22$	$0^s 91$	$18^s 0$	$22^s 7$
Pos. Med.	1834.0	$1^s 12$	$0^s 92$	$15^s 3$	$18^s 3$
Paris	67.4	$0^s 94$	$0^s 84$	$15^s 0$	$16^s 5$
Leiden	71.2	$1^s 00$	$0^s 91$	$17^s 3$	$18^s 6$

P.M. of A $-0''.036$ $+0''.045$

The relative motion of B to A from micrometer measures :

$+0''.062$ $+0''.079$
P.M. of B $+0''.026$ $+0''.124$

Catalogue.	Epoch.	R.A. 1900'0.	P.M. applied.	N.P.D. 1900'0.	P.M. applied.
Σ 2185. Mag. 7'0, 10'0, 7'7.					
		$17^h 29^m$ s		$83^\circ 55'$	
W. B. ...	1822'5	55'23	52'52	74'1	46'2
Munich ...	61'6	54'18	'84	60'7	46'9
Brussels ...	71'1	53'65	'64	..	...
Glasgow ...	79'0, 77'4	53'50	71	53'5	45'4
Romberg ...	80'4	53'47	'78	51'9	44'8
Leipzig II.	84'4	53'33	'78	51'0	45'4
Cincinnati	90'3	53'18	'84	50'0	46'5

P.M. of C $-0''\cdot522$ $-0''\cdot360$

		$17^h 29^m$ s		$83^\circ 55'$	
Lalande ...	1794'6	55'62	55'36	22'5	27'6
W. B. ...	1825	55'38	'19	21'9	25'6
Pos. Med.	42'4	55'30	'16	24'3	27'1
Munich ...	61'6	55'30	'20	23'2	25'1
Brussels ...	70'0, 70'8	55'06	54'99	25'1	26'5
Paris ...	76'5	55'01	'95	25'1	26'2
Glasgow ...	79'3	55'13	55'08	26'0	26'9
Leipzig II.	84'4	55'12	'08	25'3	26'0
Cincinnati	90'3	55'20	'18	26'6	27'0

P.M. of A $-0''\cdot037$ $+0''\cdot049$

The micrometer measures give no appreciable motion between the stars A and B, so they probably have the same P.M. Now

P.M. of C $-0''\cdot522$ $-0''\cdot360$

P.M. of A $-0''\cdot037$ $+0''\cdot049$

Hence relative motion of C to A from computed P.M.'s is

$-0''\cdot485$ $-0''\cdot409$

The relative motion of C to A from micrometer measures :

$-0''\cdot580$ $-0''\cdot488$

Σ 2514. Mag. 9'0, 11'3.

		$19^h 16^m$ s		$22^\circ 29^m$	
Pos. Med.	1833'0	49'86	...	19'1	22'5
Christiania	75'5	49'68	...	23'1	24'4
Greenwich	99'6	49'83	...	22'5	22'5

P.M. of A $0''\cdot000$ $+0''\cdot052$

The relative motion of B to A from micrometer measures :

$+0''\cdot024$ $-0''\cdot122$

P.M. of B $+0''\cdot024$ $-0''\cdot070$

Catalogue.	Epoch.	R.A. 1900.0.	P.M. applied.	N.P.D. 1900.0.	P.M. applied.
Σ 2515. Mag. 8.0, 9.0.					
		19 ^h 20 ^m s		68° 40'	
Lalande ...	1794.7	14.71	14.80	53.6	48.2
W. B. ...	1825.6	14.46	.53	49.2	45.4
Pos. Med.	28.6	14.87	.93	54.3	50.7
Berlin B.	81.6	14.93	.95	51.6	50.7

P.M. of A +0''.013 —0''.051

The relative motion of B to A from micrometer measures :

—0''.012 +0''.109

P.M. of B 0''.000 +0''.058

Σ 2658. Mag. 7.0, 9.1, 10.1.

		20 ^h 11 ^m s	s	37° 11'	
Lal. F. ...	1790	0.89	1.58	17.0	59.6
Groombridge ...	1811.7	0.66	.22	15.2	0.3
Pos. Med.	24.7	0.89	.37	14.4	1.8
Cambridge Obs. ...	44.7, 43.2	1.29	.65	10.4	0.8
Radeliffe ...	45.4, 45.7	1.05	.40	10.0	0.9
Brussels ...	71.4, 65.8	1.12	.30	6.3	0.6
Cambridge U.S. ...	77.3	1.24	.38	4.4	0.5
Cincinnati ...	95.2	1.17	.21	2.6	1.1
Greenwich ...	1900.5	1.36	.36	0.6	0.6

P.M. of A +0''.058 —0''.168

The relative motion of B to A from micrometer measures :

+0''.005 —0''.003

Hence B as the same P.M. as A.

The relative motion of C to A from micrometer measures :

—0''.032 +0''.136

Hence P.M. of C is small.

Σ 2734. Mag. 8.2, 8.7.

		20 ^h 49 ^m s	s	77° 16'	
Pos. Med.	1828.2	18.28	18.48	30.3	30.5
Leipzig I.	82.3	18.43	18.48	30.5	30.5

P.M. of A +0''.040 +0''.003

The relative motion of B to A from micrometer measures :

—0''.090 —0''.070

P.M. of A —0''.050 —0''.067

Catalogue.	Epoch.	R.A. 1900'0.	P.M. applied.	N.P.D. 1900 0.	P.M. applied.
Σ 2865. Mag. 8.5, 9.0.					
		22 ^h 2 ^m 8		20° 16'	
Lalande F. ...	1789.7	10° 52	11° 90	31° 3	29° 1
Pos. Med. ...	1823.6	10° 62	° 57	29° 6	28° 1
Oeltz. Arg. (N.)...	41.6	10° 50	° 23	31° 8	30° 6
Brussels ...	67.3, 67.8	11° 17	° 58	28° 7	28° 1
Christiania ...	75.5	11° 12	° 43	27° 6	27° 1
Greenwich ...	99.2	11° 52	° 53	28° 2	28° 2

P.M. of A + 0''·064 — 0''·020

The relative motion of B to A from micrometer measures :

— 0''·072 + 0''·054
P.M. of B — 0''·008 + 0''·034

Note on the Determinations of Positions and Magnitude of Stars in the Greenwich Astrographic Catalogue.

(Communicated by the Astronomer-Royal.)

The Introduction to vol. i. of the Greenwich Section of the *Astrographic Catalogue*, which is now in the press and will shortly be published, contains short discussions of the personality of the measurers, the probable error of the measures and of resulting right ascensions and declinations, and of the relation between magnitude of stars and the diameters of their photographic images. It may be of interest to lay before the Society a brief summary of these investigations.

I. Personality of Measurers.

The duplicate measurement of the plates in reversed positions, with a view to an increase of accuracy, was undertaken as a result of the meeting of the Astrographic Committee at Paris in 1896 June. Zones 64°, 65°, 66°, 67°, had by this time been measured at Greenwich in the direct positions of the plates. They were accordingly re-measured, the plates and the glass diaphragm being reversed right for left. The direct and reversed measures were compared, the investigation involving many thousands of measures of about 300 plates by eight different measurers. In the direct measures the same measurer measured both the 6^m and 3^m images, but in the reversed measures there were separate measurers for the two images.

The means of the measures of the 6^m and 3^m images having been already formed, the differences of these means direct and reversed were tabulated for each combination of measurers, and for convenience each book of measures was dealt with separately. As an example the following table is given, showing the result of the collection of these differences for the second book of zone 66°, extending from R.A. 6^h 0^m to 11^h 24^m. It should be stated that the "direct" measures of this book were made in 1896 March and the "reversed" measures in 1897 April.

Differences of Measures Direct-Reversed as made by different Observers for Zone 66° R.A. 6^h 0^m—11^h 24^m.

Unit = 0001 of a réseau interval = ''03.

Measurers.				Centre +66° (South halves).				Centre +67° (North halves).			
Direct.		Reversed.		No. of Stars.	Sum of Diff.	No. of Stars.	Sum of Diff.	No. of Stars.	Sum of Diff.	No. of Stars.	Sum of Diff.
6 ⁿ	3 ^m	6 ⁿ	3 ^m								
J.,	J.	J.,	P. M.	36	+ 536	39	+ 288	59	+ 1214	59	+ 273
J.,	J.	W. S.,	P. M.	6	+ 35	6	+ 76	8	+ 96	8	+ 33
J.,	J.	C. D.,	P. M.	15	+ 142	15	+ 792	29	+ 216	29	- 49
W. S.,	W. S.	J.,	P. M.	32	+ 249	32	- 219	57	- 286	57	- 1
W. S.,	W. S.	J.,	E. S.	37	+ 192	37	- 292	29	- 145	29	- 35
W. S.,	W. S.	W. S.,	P. M.	67	+ 316	65	- 672	67	- 824	68	- 76
W. S.,	W. S.	W. S.,	E. S.	102	+ 692	102	- 1087	104	- 1503	107	+ 669
W. S.,	W. S.	P. M.,	E. S.	16	+ 62	16	- 96	17	- 199	18	+ 72
W. S.,	W. S.	C. D.,	J.	18	+ 132	19	- 221	19	+ 65	20	- 21
P. M.,	P. M.	J.,	W. S.	29	- 90	27	+ 21	38	- 202	36	+ 27
P. M.,	P. M.	J.,	P. M.	18	- 198	19	+ 32	55	- 330	55	+ 261
P. M.,	P. M.	W. S.,	E. S.	50	- 293	49	- 525	47	- 561	47	+ 290
P. M.,	P. M.	C. D.,	P. M.	23	- 286	23	- 71	19	- 264	19	+ 27
E. S.,	E. S.	W. S.,	E. S.	53	+ 53	52	- 526	61	- 680	62	- 701
E. S.,	E. S.	C. D.,	P. M.	3	- 8	3	- 21	4	+ 47	4	- 38
E. S.,	E. S.	J.,	W. S.	7	+ 41	7	- 103	10	- 57	10	- 86
E. S.,	E. S.	J.,	P. M.	31	+ 205	31	- 57	30	+ 5	29	- 378
E. S.,	E. S.	J.,	E. S.	20	+ 89	20	- 69	25	+ 48	25	- 8
Total No. of Stars ...				563		562		678		682	

Denoting by the subscripts 1 and 2 measures made in the direct and reversed positions respectively, the above table gives equations of condition of the form

$$36\left(J_{.1} - \frac{J_{.2} + P.M._2}{2}\right) = +536$$

Normal equations were formed and solved in the usual way. The zero being arbitrary, it was assumed so that $J_{.1} + J_{.2} + W.S._1$

+ W.S.₂=0, as J. and W.S. had measured for a long period. The solution of these equations gave the following values for the personalities of the observers, in which a distinction is made according to the microscope of the duplex micrometer used :

	J. ₁	J. ₂	W.S. ₁	W.S. ₂	P.M. ₁	P.M. ₂	E.S. ₁	E.S. ₂
<i>x</i> Left-hand microscope	+ 9	- 7	+ 3	- 4	- 10	+ 1	- 2	0
<i>x</i> Right-hand microscope	+ 16	- 11	- 10	+ 6	- 9	+ 2	- 5	+ 3
<i>y</i> Left-hand microscope	+ 6	- 5	- 8	+ 5	- 4	- 1	- 6	+ 4
<i>y</i> Right-hand microscope	+ 1	- 1	- 5	+ 4	+ 3	- 7	- 11	- 7

Nineteen books of measures were treated in this manner, and the results, which are given in the Introduction to the Greenwich Astrographic Catalogue, show that these personalities were fairly constant. The mean results (with changed signs) are given in the following table of corrections. Different measures show clearly that the mean of the corrections, direct and reversed, is nearly zero for each measurer; that is to say, the mean of a direct and reversed measure by the same measurer is almost free from personality.

Corrections for Personality.

$$\text{Unit} = \text{int.}^{\circ}0001 = 0''\cdot03.$$

Measurer.	Measures of <i>x</i> Coordinate.				Measures of <i>y</i> Coordinate.	
	Right-hand Microscope. Direct.	Reversed.	Left-hand Microscope. Direct.	Reversed.	Both Microscopes. Direct.	Reversed.
Miss Everett (A.E.)	+ 3	...	+ 1	...	+ 5	...
Mr. Davidson (C.D.)	+ 5	+ 2	+ 3	+ 1	+ 6	- 3
Mr. Johns (J.)	... - 11	+ 8	- 9	+ 7	+ 1	+ 1
Mr. Stevens (W.S.)	+ 6	- 5	- 2	+ 3	+ 6	- 5
Mr. Melotte (P.M.)	+ 3	- 2	+ 3	- 1	- 4 or + 2*	+ 4 or - 2*
Mr. Skells (E.S.)	... + 3	- 7	+ 2	+ 2	0	+ 4
Mr. Evans (E.)	... + 1	- 4	...	+ 9	0	0
Mr. Stiles (St.)	... + 13	- 9	0	- 12	- 4	- 5

As a result of this investigation it was arranged that the same measurer should measure the 6^m images in the direct and reversed positions of the plate, and another measurer the 3^m images also in both positions.

II. *Probable Error of the Measures.*

In 1896 July two investigations were made of the probable error of the measures, including the errors inherent in the photographic images of a star and the réseau lines to which it is referred.

* The values + 2 and - 2 were found for some measures made (direct and reversed) at a later date in 1897 June-August.

(i.) Measures of the same plate were made by five measurers, each measurer measuring both the 6^m and 3^m images on the plate in direct and reversed positions.

(ii.) Five plates of the same field taken at different times and under different conditions were measured by one measurer in direct and reversed positions, giving ten measures.

(i.) The values for the discordance of the mean of the measures of a 6^m and 3^m image by one observer in one position (direct or reversed), corrected for personality, from the mean of ten such measures of 113 stars was found to be

Measurer.	<i>x</i> Coordinate.		<i>y</i> Coordinate.	
	Direct.	Reversed.	Direct.	Reversed.
Mr. Davidson ...	± "120	± "135	± "150	± "195
Mr. Johns ...	± '219	± '186	± '219	± '174
Mr. Stevens ...	± '177	± '174	± '171	± '159
Mr. Melotte ...	± '231	± '222	± '219	± '219
Mr. Skells ...	± '213	± '186	± '186	± '183

The means of the above discordances are $\pm''\cdot186$ and $\pm''\cdot187$ in the two coordinates respectively, leading in each case (after multiplication by $\sqrt{\frac{1}{9} \times \cdot845}$) to a probable error of $\pm''\cdot166$. Thus the probable error of a complete measure (two images, direct and reversed) is $\pm''\cdot117$.

It is to be noted that a complete measure involves eight readings or estimations of the positions of the two réseau lines relatively to the star images, these being practically quite independent, as owing to irregular curvature of the plate the "run" for a réseau interval varies from point to point.

If, instead of applying a correction for personality, the means of the measures, direct and reversed, are taken for each observer, the discordances are found to be

Measurer.	<i>x</i> Coordinate.			<i>y</i> Coordinate.
Mr. Davidson ...	...	...	± "108	± "131
Mr. Johns ...	...	...	± '153	± '159
Mr. Stevens ...	...	...	± '114	± '126
Mr. Melotte ...	...	...	± '132	± '141
Mr Skells ...	...	...	± '144	± '132

The probable error of a complete measure (multiplying the mean discordance by $\sqrt{\frac{1}{4} \times \cdot845}$) is found to be $\pm''\cdot127$. This agrees satisfactorily with the result given above, showing that personality of the measurer is virtually eliminated in the mean of measures direct and reversed.

(ii.) Five plates of the same field were completely measured by Mr. Stevens, direct and reversed. In each case the means of the measures of the 6^m and 3^m images were taken. The plate

constants were obtained exactly as is done in practice by comparing the measured coordinates with the standard coordinates of the reference stars of which there were twenty-two on the plate. Corrections of the form $ax+by+c$, $dx+ey+f$ were applied to the measured coordinates, thus giving ten comparable values of the coordinates of 121 stars. The means of the ten measures, direct or reversed, were taken for each star, and the discordances from the mean formed. The mean discordances are given in the following table :

Plate No.	x Coordinate.		y Coordinate.	
	Direct.	Reversed.	Direct.	Reversed.
419	$\pm''42$	$\pm''35$	$\pm''34$	$\pm''31$
3139	$\pm'25$	$\pm'35$	$\pm'20$	$\pm'29$
3150	$\pm'34$	$\pm'26$	$\pm'19$	$\pm'24$
3151	$\pm'28$	$\pm'25$	$\pm'21$	$\pm'36$
3089	$\pm'27$	$\pm'29$	$\pm'26$	$\pm'24$

The mean of these discordances is $\pm''285$, giving $\pm''254$ for the probable error of a single determination of position (6^m and 3^m image) and $\pm''180$ of a complete determination, direct and reversed.

If the discordances of the means of the direct and reversed measures are taken for the five plates, the results are :

Plate No.	x Coordinate.	y Coordinate.
419	$\pm''34$	$\pm''30$
3139	$\pm'18$	$\pm'17$
3150	$\pm'27$	$\pm'20$
3151	$\pm'20$	$\pm'23$
3089	$\pm'25$	$\pm'21$

and the probable error of a complete measure deduced in this way is $\pm''222$. This increase in the probable error may be taken as due to the systematic error which is common to the plate, both for direct and reversed measures.

It should be noted that in this probable error are included (i.) the errors of the measures, (ii.) the errors inherent in the photographic images of stars and réseau lines, (iii.) the errors introduced through errors in the plate constants arising from these causes, but not the part arising from the assumed right ascensions and declinations of the reference stars.

A further discussion of probable errors of places of the stars deduced from the photographs has been made by comparing the separate results from pairs of plates for a limited number of stars. The right ascensions and declinations of 240 stars between 70° N.Dec. and 72° N.Dec. have been deduced, with the provisional plate constants given in the Introduction to the Catalogue, one

star being taken in each quarter of the overlapping plates in this zone. A comparison of the results from the two plates on which each star was shown was made, the mean differences being found to be $\pm''\cdot43$ in R.A. and $\pm''\cdot46$ in Dec. From this it results that the probable errors (in arc of a great circle) of the right ascension and declination of a star deduced from the measures on one plate are $\pm''\cdot26$ in R.A. and $\pm''\cdot28$ in Dec., and that the probable errors of a catalogue place from the mean of two plates may be taken as $\pm''\cdot19$ in both R.A. and declination. These results were obtained with provisional places of the reference stars, the accidental errors of which affect the plate constants to some extent. In order to obtain more accurate places of the reference stars for determination of plate constants, these stars are being re-observed with the transit-circle at Greenwich; and when these observations are completed in the course of a couple of years the accuracy of the plate constants will be materially increased. From a discussion of the observations of the reference stars from N.P.D. 0° to 5° , now completed, it appears that the probable error of a star's place in the new Greenwich Meridian Catalogue (five observations in each element) is $\pm 0''\cdot23$ in R.A. or N.P.D.

III. *Determination of Photographic Magnitudes.*

In the publication of the Greenwich Measures the diameter of the 6^m image is given as a measure of the photographic magnitude. In each case the diameter is estimated by two measurers in divisions and tenths of a division of the scale and the sum taken. Allowance is made for images which are sensibly elongated by taking the mean of the diameters in the horizontal and vertical directions. Allowance is also made by the measurers for the want of density in the images and also for the shading off at the edge, and the diameter set down is that of an equivalent black image, *i.e.* one which would contain the same amount of silver deposit.

In order to obtain direct information as to the magnitude of the faint stars shown on the plates, comparison has been made with the determinations on a photometric scale given by Professor E. C. Pickering (*Harvard Annals*, vol. xxxvii. pp. 4-11) of the faint stars near certain variables which lie in the Greenwich section. The photometric magnitudes of faint stars near the pole (*ibid.*, vol. xviii. p. 134) have also been compared with the diameters of the images on a catalogue plate (exp. 6^m , 3^m , and 20^s) and on four chart plates (exp. 40^m). Some photographs of *Nova Persei* taken under the same conditions as the astrographic plates also admit of comparison with the magnitudes given by Father Hagen (Georgetown College Observatory, *Supplementary Notes to the Atlas Stellarum Variabilium*).

The following table, giving the stars near *R Draconis* R.A. $16^h\ 32^m\ 23^s$, Decl. $+66^\circ\ 57'\cdot8$ (1900·0), shown on plates with

centres at R.A. $16^h 30^m$, Decl. $+67^\circ$, may serve as a specimen of the comparisons made, which deal with 19 chart plates and 18 catalogue plates of seven different fields.

Diameters of Images of Stars near R Draconis.

Unit = $0''.15$.

Reference Letter.	Photom. Mag.	Plate 2652. Exp. 20".	Plate 2652. Exp. 6".	Plate 3482. Exp. 40".	Zone and No. in Greenwich Measures.
<i>a</i>	6.88	29	74	121	67, 5303
<i>c</i>	7.44	28	70	117	67, 5299
<i>f</i>	8.28	14	43	82	66, 5250
<i>g</i>	8.56	20	51	84	67, 5395
<i>h</i>	9.06	12	42	80	67, 5283
<i>k</i>	9.40	7	32	62	66, 5248
<i>l</i>	9.80	6	20	52	67, 5317
<i>m</i>	10.16	5	28	54	67, 5370
<i>n</i>	10.56	5	24	46	66, 5223
<i>o</i>	10.97	not shown	15	34	66, 5222
<i>p</i>	11.59	"	9	26	66, 5251
<i>q</i>	12.01	"	8	20	66, 5234
<i>r</i>	12.52	"	6	20	66, 5254
<i>s</i>	12.85	"	not shown	18	...
<i>t</i>	13.33	"	"	13	...
<i>u</i>	13.84	"	"	15	...
<i>w</i>	14.43	"	"	13	...
<i>x</i>	15.02	"	"	10	...

The faintest stars shown with an exposure of 6^m on all the plates where a comparison with photometric magnitudes could be made are shown in the following table :—

Plate.	Mag. m	Plate.	Mag. m	Plate.	Mag. m
4061	12.61	4027	12.92	6160	11.6
1678	11.08	2652	12.52	6168	12.7
4767	11.87	2660	12.01	6178	11.9
3409	12.02	2776	11.96	6200	11.8
2469	12.02	2771	11.96	6202	12.8
2708	12.92	5013	13.2	6204	12.7

The mean for the above plates is $12^m.25$, and this may be taken as an approximate value of the average limit of magnitude reached by the catalogue plates at Greenwich with an exposure of 6 minutes.

For the conversion of diameters into magnitudes the em-

pirical law connecting the measured diameter d and the magnitude m determined by the Astronomer Royal (*Monthly Notices*, vol. lii. pp. 125-146), viz.:— $m = C - n\sqrt{d}$, has been adopted and appears to give a satisfactory accordance with the photometric determinations. The following determinations of the values of the constants C and n are derived by direct comparison of the bright and faint stars on each of 19 chart plates (exp. 40^m):—

Plate.	Field.	Range of magnitudes and No. of stars.			Deduced formula $m=C-n\sqrt{d}$.	Diameter given by formula.					
		m	m			9 ^{m.0}	11 ^{m.0}	13 ^{m.0}	14 ^{m.0}	15 ^{m.0}	
1517	S. Cassiop.	8.76	1	13.93	3	15.7-0.68 $\sqrt{d}$	97	48	16	6	1
5800	„	8.76	1	13.93	3	15.7-0.68 $\sqrt{d}$	97	48	16	6	1
3719	„	9.92	2	14.12	2	16.0-0.75 $\sqrt{d}$	87	44	16	7	2
3878	R. Urs. Maj.	8.49	2	14.01	3	16.7-0.74 $\sqrt{d}$	109	59	25	13	5
3436	„	7.94	2	14.01	3	15.8-0.85 $\sqrt{d}$	64	32	11	5	1
4019	R. Urs. Min.	8.76	4	12.55	3	16.6-0.90 $\sqrt{d}$	71	40	16	8	3
3515	„	8.94	1	12.55	3	16.1-0.87 $\sqrt{d}$	67	35	13	6	2
3469	R. Draconis	8.91	3	14.43	3	17.2-1.00 $\sqrt{d}$	67	38	18	10	5
3482	„	7.16	2	14.43	3	17.6-1.00 $\sqrt{d}$	74	44	21	13	7
4587	T Cephei	8.46	2	12.55	2	16.4-0.87 $\sqrt{d}$	74	40	16	8	3
3223	„	8.12	1	12.55	2	16.3-0.81 $\sqrt{d}$	82	43	17	8	3
6058	Pole	10.00	4	14.40	4	16.6-0.95 $\sqrt{d}$	64	35	15	7	3
6050	„	10.00	4	14.40	4	16.2-0.78 $\sqrt{d}$	85	45	17	8	2
5955	„	10.00	4	14.40	4	16.1-0.95 $\sqrt{d}$	56	29	11	5	1
5964	„	10.00	4	14.40	4	16.4-0.98 $\sqrt{d}$	57	31	12	6	2
6200	Nova Persei	7.70	5	12.50	9	14.6-0.77 $\sqrt{d}$	53	22	4	1	...
6202	„	7.70	5	12.50	9	16.0-0.79 $\sqrt{d}$	79	40	14	6	2
6204	„	7.70	5	12.50	9	15.2-0.84 $\sqrt{d}$	54	25	7	2	1
6161	„	7.70	5	12.50	9	14.7-0.78 $\sqrt{d}$	53	22	5	1	...

The mean formula derived from the chart plates (exposure 40^m) is $m = 16.1 - 0.84\sqrt{d}$.

Formulae were derived in a similar way for the exposures of 6^m , 3^m , and 20^s of the catalogue plates, the mean results being as follows:—

$$\begin{array}{l}
 m \\
 \left. \begin{array}{l}
 m = 16.1 - 0.84\sqrt{d} \text{ for exposure } 40^m \text{ (19 plates)} \\
 m = 13.7 - 0.77\sqrt{d} \text{ „ „ } 6^m \text{ (18 plates)} \\
 m = 12.9 - 0.76\sqrt{d} \text{ „ „ } 3^m \text{ (6 plates)} \\
 m = 11.5 - 0.79\sqrt{d} \text{ „ „ } 20^s \text{ (9 plates)}
 \end{array} \right\}
 \end{array}$$

Putting $d=4$ as practically the diameter of a star which is shown quite clearly (though there are a number of stars measured with diameters 3 or 2), we find

$m=14.4$	for the average limiting magnitude with exposure	40^m
$m=12.3$	"	6^m
$m=11.4$	"	3^m
$m=9.9$	"	20^s

The following table gives the observed difference of magnitude in passing from one exposure to another compared with the values of $2.5 \log t_2/t_1$:

	Obs. Diff.	$2.5 \log \frac{t_2}{t_1}$
	m	m
20^s to 3^m	1.5	2.39
3^m to 6^m	0.9	0.75
6^m to 40^m	2.1	2.06
20^s to 40^m	4.5	5.20

It appears from this that in passing from 3^m to 6^m , and again from 6^m to 40^m , the law

$$\text{Exposure} \times \text{brightness} = \text{constant}$$

holds almost exactly, while the exposure of 20^s gives fainter stars than would be expected in accordance with the law. As a possible explanation of this it is to be noted that the 20^s exposure was given after the exposures of 6^m and 3^m , and that this preliminary exposure to diffused light may have increased the sensitiveness of the plate for the 20^s exposure.

In a paper on the statistics of stars in a zone of 5° from $+65^\circ$ to $+70^\circ$ Dec. (*Monthly Notices*, 1903 January), the logarithms of the number of stars per square degree with different exposures are given as follows:—

I. *Shown on both plates.*

20^s	3^m	6^m	40^m
0.966	1.598	1.724	2.437

II. *Total number shown.*

20^s	3^m	6^m	40^m
1.186	1.818	1.830*	2.496*

Between the 20^s and 3^m exposure the log increase of number

* As stated in the above paper the number 1.830 is too small, as stars which show a 6^m image on one plate only are not counted. The number 2.496 is probably too small, though not to the same extent.

of stars per square degree is 0.632 or (the difference of magnitude being 1.5) at the rate of 0.42 per magnitude.

Between 20^s and 6^m the log increase of number of stars per square degree is 0.757 , or (the difference of magnitude being 2.4) at the rate of 0.32 per magnitude.

Taking the mean of these results we find that between the magnitudes $10^{m.0}$ and $12^{m.0}$ the logarithmic increase per magnitude in the total number of stars is 0.37 , and therefore the ratio of increase per magnitude is 2.34 .

Between 3^m and 40^m there is a logarithmic increase of 0.839 , corresponding to a difference of magnitude of $3^{m.0}$ or 0.28 per magnitude, and between 6^m and 40^m of $.713$, corresponding to a difference of magnitude of $2^{m.1}$ or of 0.34 per magnitude.

Taking the mean of these results we have between $12^{m.0}$ and $14^{m.4}$ a logarithmic increase of number of stars per magnitude $= 0.31$, corresponding to a ratio of increase of 2.04 per magnitude.

It is satisfactory to find that these results are in good accordance with those recently given by Professor Pickering in his valuable memoir on the "Distribution of Stars" (*Harvard Annals*, vol. xlviii. No. 5, p. 178). Taking the values given in his Table XXI., the logarithmic increase per magnitude in the total number of stars would be 0.39 (as compared with 0.37 found above) between 10 and 12 mag., and 0.31 (as compared with 0.31) between $12^{m.0}$ and $14^{m.4}$, the corresponding ratios of increase per magnitude in the total number of stars being 2.45 and 2.04 found by Professor Pickering, as compared with 2.34 and 2.04 found above.

Royal Observatory, Greenwich :
1904 March 8.

On the Chromatic Correction of Object-glasses. Second Paper.
By A. E. Conrady.

In my first paper * I limited myself to the consideration of a luminous point in the optical axis. Precisely the same reasoning may, however, be applied to any point in the field of an object glass; but as in the case of oblique pencils there is no such complete symmetry as in axial pencils, a greater number of rays have to be followed through the system of lenses and their paths in the successive media determined in order to compare the value of ΣnD for the several rays, and thus to determine the state of the chromatic correction.

It will generally be sufficient to compute four rays through the system: three of these lie in the plane containing the optical axis and the luminous point; these should be computed

* *Monthly Notices*, January 1904.

by the trigonometrical formulæ given in my previous paper. They are those rays in that plane which pass through the centre and margins of the limiting aperture of the system. The fourth ray passes through the margin of the same aperture vertically above or below the central ray; it does not cut the optical axis and must be computed by formulæ adapted to this case; probably those given by von Seidel* are the most convenient. The paths of each of these four rays in the several constituent lenses of the system can then be computed by the formula given in my previous paper (*l.c.* page 187). In the case of the fourth ray, and adopting the nomenclature used in Steinheil and Voit's reprint of Seidel's classical paper, the angle at the centre of curvature, which I call α , is there given as η , and the angle between the ray and a parallel to the optical axis, which takes the place of my angle β' , is also given as τ' , so that my formula for D may be applied with the same facility to this ray as to those proceeding in planes containing the optical axis.

Then, if $\Sigma dn \cdot D$ has exactly the same value for all four rays, the system will be perfectly achromatic for the luminous point chosen and for the selected wave-length. But the values of the sum will generally, owing to the extreme difficulty of perfect correction for oblique pencils, differ more or less from each other. Let $\Sigma dn D_0$ be the value of the sum for the central ray, $\Sigma dn D_1$ and $\Sigma dn D_2$ the respective values for the marginal rays in the plane containing the optical axis, and $\Sigma dn D_3$ the value for the marginal rays farthest from that plane; then it is not difficult to see that:

1. If $\Sigma dn D_1 < \Sigma dn D_2$ the refracted coloured waves will form an angle with, and will therefore proceed in a direction differing from, that of the refracted principal waves; in other words, the object-glass will produce different-sized images for different colours, or the image of the luminous point will be drawn out into a spectrum.

2. If $\Sigma dn D_1 = \Sigma dn D_2$ but $> \Sigma dn D_0$, the coloured and principal waves will have the same direction, but different curvature and therefore different foci, and the image of the luminous point will be affected with symmetrically arranged coloured fringes.

3. If $\Sigma dn D_3$ differs from $\frac{1}{2}(\Sigma dn D_1 + \Sigma dn D_2)$ there will be an astigmatic difference for different colours, and similar considerations will lead to a correct interpretation of any other case that may be met with.

The reasoning employed in my first paper may also be applied to prisms, for there is nothing in the train of thoughts which leads to my expression for the chromatic aberration of a system of lenses which cannot be equally well applied to plane or curved waves proceeding through a train of prisms; hence if d

* "Sitzungsberichte der math.-phys. Classe der kgl. bayr. Akademie der Wissenschaften vom 10. Nov. 1866," reprinted in Steinheil and Voit, *Handbuch der angewandten Optik*, vol. i., Leipzig, 1891.

and D are taken as the respective paths of the two marginal rays transmitted, $\Sigma dn(d-D)$ expresses the differential dispersion of the train of prisms, measured as the distance between the refracted principal and coloured waves on one ray when they intersect on the other; and accepting the postulate that this distance must be equal to the wave-length in order to give sufficiently separated foci of the two waves, we obtain at once Lord Rayleigh's famous formula for the resolving power of spectroscopes,* at which the great physicist arrived by a totally different path.

Conversely, my formula as applied to lenses may be interpreted in accordance with the familiar treatment of lenses as prisms of variable angle as demanding that the total resolving power of the system shall be zero for the selected wave-length, and so stated it has an almost axiomatic appearance.

With regard to suitable formulæ for determining the refractive and dispersive powers of optical glass for any wave-length, I have repeated the necessary calculations, with the result that the statements in my first paper are amply confirmed; it would indeed seem that the formula there given as the best:

$$n_{\lambda} = n_0 + \nu_1 \lambda^{-\frac{2}{3}} + \nu_2 \lambda^{-4}$$

is even more accurate than I stated, and will generally represent the indices of any ordinary silicate glass within the visible spectrum to less than one unit of the fifth decimal if well selected lines are chosen for determining the constants.

These calculations led to the discovery of yet another empirical formula which is slightly more convenient and at least equally accurate, and should therefore be preferable, viz.

$$n_{\lambda} = n_0 + \nu_1 \lambda^{-1} + \nu_2 \lambda^{-3\frac{1}{2}}$$

Finally, an application of the various formulæ to Langley's determination of the refractive indices for a light flint prism from $\lambda = 20,300$ to $\lambda = 3440$ (O line) had the interesting result of showing that the empirical formulæ retain their superiority throughout that range, for when the constants were determined from the indices for the A, F, and O lines, Cauchy's formula (with three terms) erred by twice as much for the longest wave-length as any of the others, and for other wave-lengths the superiority of the empirical formulæ proved even greater.

* *Encyc. Brit.*, 9th edition, vol. xvii. page 807. Also "Investigations in Optics," *Phil. Mag.* 1879, 1880.

On the Determination of the Division Errors of a Graduated Circle. By S. S. Hough, M.A., F.R.S.

§ 1. *Introductory.*

The accurate determination of the division errors of a graduated circle can only be made at the expense of a considerable amount of labour, and it becomes a matter of the highest importance at the outset of such an undertaking to investigate how the available appliances may be utilised to the highest advantage with the least possible work. The considerations dealt with in the following paper have arisen in the course of the preparation of a programme for the determination of the errors of divisions of the new transit circle of the Cape Observatory now in progress, and it will serve to coordinate ideas if we deal with the problem from the outset in the form in which it is presented by that instrument. The subsidiary appliances will, of course, vary with different instruments, but those attached to the Cape instrument may at least be taken as sufficiently typical, while the principles involved may be readily adapted to other instruments and appliances.

The circles of the Cape instrument are ordinarily read by means of six microscopes symmetrically situated round the centres. For the determination of errors of graduation the instrument is also provided with two pairs of subsidiary microscopes, situated each at extremities of a diameter at angular distances of 20° and 25° respectively from one pair of the primary six.

The six primary microscopes enable us to determine the relative errors of six equi-spaced divisions with a high degree of accuracy, and the subsidiary microscopes then furnish the means of determining the errors of division—first, of division marks separated from the former by an interval of 20° ; and, secondly, of those separated by an interval of 25° —*i.e.* we can ascertain the relative spacing of division marks separated by intervals of 5° throughout the circle. A further appliance, consisting of a microscope provided with a divided object glass similar to that used in the determination of the division errors of the old Cape transit circle,* enables us still further to subdivide the spaces of 5° into spaces of 1° interval.

We proceed first to consider the geometrical significance of a single pointing made by means of one of the micrometers on a division mark.

§ 2. *Derivation of the Fundamental Equation.*

Assume, in the first instance, that the circles are perfectly plane and perfectly rigid, and that the division marks consist of approximately radial straight lines engraved on them.

* Introduction to *Cape Observations*, 1879–81.

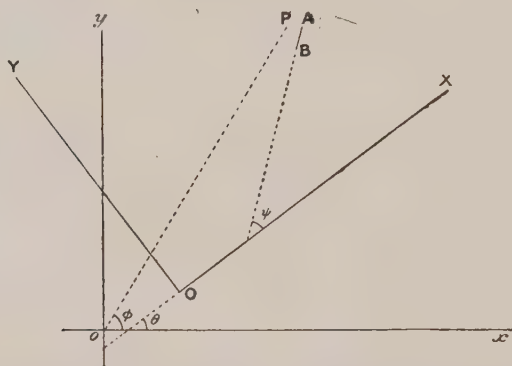
Take a pair of rectangular axes OX , OY fixed relatively to a circle. The origin O is supposed to be at the centre of the circle, and the direction OX directed towards the division mark 0° . Departures from these conditions of the order of magnitude of the instrumental errors may, however, be supposed to exist, and the precise definition of the point O and the direction OX with relation to these errors is deferred.

With reference to these axes the position of a division mark, assumed to be a straight line, may be represented by an equation of the form

$$X \sin \psi - Y \cos \psi = p,$$

where p will be a small quantity of the order of magnitude of the instrumental errors, and for the division mark marked n° the angle ψ will differ from n° by a like small amount.

Now suppose that the circle under investigation is mounted in such a manner as to allow freedom of motion in its own plane



without variation of this plane in space, and that it can be viewed by a system of microscopes provided with micrometers, which are themselves regarded as *fixed in space*.

The operation of bisection consists in placing a pair of close parallel webs, movable by means of the micrometer screw, in the position symmetrical with respect to a division mark; but, in case the division mark should not be strictly parallel to the moving webs, the bisection is always made between two fixed parallel webs at right angles to the moving webs and somewhat more widely separated than they.

If, then, a micrometer be set to its zero reading, the intersection of the pair of straight lines lying midway between the fixed and moving webs will define a point fixed in the focal plane of the microscope, which is conjugate to a fixed point in the plane of the circle. Thus the zero reading of the micrometer will define a point fixed in the plane of the circle, and the reading when the micrometer is pointed on any division mark

will measure in terms of the micrometer screw the perpendicular distance of this point from the division mark regarded as a straight line.

Now take a pair of fixed rectangular axes ox, oy in the plane of the circle, and let ξ, η denote the coordinates of the origin O , fixed relatively to the circle, with reference to the axes ox, oy fixed in space; further, let θ denote the angle between the two axes OX, Ox .

Let $R \cos \phi, R \sin \phi$ be the coordinates relatively to the fixed axes ox, oy of the fixed point associated with one of the micrometers, so that R, ϕ will denote two "constants of the microscope."

In the opposite figure this point is denoted by P and the division mark by the straight line AB .

On referring P to the moving axes OX, OY its coordinates will be

$$\begin{aligned} & R \cos \phi \cos \theta + R \sin \phi \sin \theta - \xi \cos \theta - \eta \sin \theta \\ & - R \cos \phi \sin \theta + R \sin \phi \cos \theta + \xi \sin \theta - \eta \cos \theta, \end{aligned}$$

or

$$R \cos (\phi - \theta) - \xi \cos \theta - \eta \sin \theta, \quad R \sin (\phi - \theta) + \xi \sin \theta - \eta \cos \theta,$$

and the perpendicular distance of this point from the division mark AB , whose equation is

$$X \sin \psi - Y \cos \psi = p,$$

will be

$$p - R \sin (\theta + \psi - \phi) - \xi \sin (\theta + \psi) + \eta \cos (\theta + \psi).$$

If then m denotes the actual reading of a micrometer head, $m(1+r)+s$ the same corrected for "run" and screw error—i.e. referred to a unit and zero defined with reference to the plane of the circle instead of with reference to the screw—every micrometer reading will yield an equation of condition of the form

$$p - R \sin (\theta + \psi - \phi) - \xi \sin (\theta + \psi) + \eta \cos (\theta + \psi) - m - mr - s = 0.$$

This equation involves three "constants of the microscope," viz. R, ϕ, r ; two "constants for the division mark," viz. p, ψ ; and three quantities, ξ, η, θ , which depend on the position of the circle beneath the microscopes, and which will be the same for all microscopes at any one setting of the circle.

Further, the equation expresses in its most general form the information which can be derived from any one pointing, provided that the geometrical conditions that we have presupposed have been fulfilled.

In practice the values of m which can occur will, of course, be limited by the field of view of the microscopes and the range of the micrometer screws; we may consequently regard $\theta + \psi - \phi$ as a small angle, and replace $\sin (\theta + \psi - \phi)$ by $\theta + \psi - \phi$.

Further, in operations for the determination of division errors

we may always suppose that the division mark is brought under the microscope in such a way as to be very approximately bisected when the micrometer is set to its zero reading. (The actual reading which may be regarded as zero can, of course, be arbitrarily selected.) Thus, if we suppose that θ_0 , ϕ_0 , ψ_0 , R_0 are the ideal values of θ , ϕ , ψ , R aimed at in the construction, adjustment, and setting of the instrument, we may suppose that

$$\theta = \theta_0 + \delta\theta, \phi = \phi_0 + \delta\phi, \psi = \psi_0 + \delta\psi, R = R_0 + \delta R,$$

where we have rigorously

$$\theta_0 + \psi_0 - \phi_0 = 0,$$

while R_0 is the same for all microscopes, and $\delta\theta$, $\delta\phi$, $\delta\psi$, δR represent small errors whose squares we may regard as negligible.

Our equation thus reduces to

$$p - R_0(\delta\theta + \delta\psi - \delta\phi) - \xi \sin \phi_0 + \eta \cos \phi_0 - m - s = 0,$$

where in the small terms involving ξ , η we have replaced $\theta + \psi$ by its approximate value ϕ_0 , and we have omitted the term m , since m will now be a small quantity of the order of magnitude of the small errors of adjustment, &c. The runs of the microscopes may easily be reduced by mechanical adjustment so as to be practically insensible over the small range of the micrometer screw which need be called into play.

If, then, we put

$$p - R_0\delta\psi = x$$

$$R_0\delta\phi + s = y$$

$$-R_0\delta\theta = z$$

we obtain

$$x + y + z - \xi \sin \phi_0 + \eta \cos \phi_0 = m,$$

where x is a quantity which depends only on the division mark bisected, and which we may describe as the division error of the mark ξ .

y is a quantity which depends only on the micrometer used. If a sufficiently small range of the micrometer screw be involved the part s may be regarded as constant, and y will then denote a constant error in adjustment of the microscope. The screw errors, if considerable or liable to rapid variation, should, however, be independently determined and applied to the micrometer readings. There will then remain a residual correction y , which is constant for the microscope.

z is a quantity which depends only on the angular position in space of the divided circle, which we shall refer to in future as the error in setting of the circle.

If the pivots were truly cylindrical it would be possible to choose the points O , o at the axis of pivots which will be fixed

both with reference to the circle and with reference to the microscopes. In this case the quantities ξ , η would each be zero, and the equation would reduce to

$$x + y + z = m.$$

The same form of equation would result if the forms of the pivots were supposed known from previous investigations, since ξ , η could then be regarded as known functions of θ , their values being applied as a correction to the absolute term m .

§ 3. *Grouping of Observations.*

The problem involved in the determination of division errors consists in the arrangement of the observations in such a manner as to allow of the elimination of the quantities y , z , ξ , η and the evaluation of the quantities x with the greatest possible weight by means of the resulting equations.

The formation of our fundamental equation rests on the assumption that the microscopes are fixed in space. In practice the adjustment of the micrometers will be liable to incessant change, on account of changes of temperature, &c. These changes may, however, be assumed to take place slowly; it will therefore be necessary that the observations should be grouped into short sets, during each of which the quantities y may be regarded as sensibly constant, and each of which will admit of the elimination of the y 's, either of itself or in combination with other similar sets. The effective constancy of the y 's during a set may be further ensured if a set consists of a series of operations immediately followed by an exactly similar series in reversed order.

The errors of the pivots will be completely eliminated from the mean of a pair of readings taken with opposite microscopes at any one setting of the circle, since the coefficients of ξ , η will have the same values with opposite signs for these two readings. Thus if we restrict our operations to the determination of the mean division errors of pairs of opposite division marks we need concern ourselves no further with the errors due to irregularity in the forms of the pivots. The restriction will be of no material consequence if in future use the circle is always read by pairs of opposite microscopes.

The only outstanding source of systematic error, at least of a geometrical character, which can affect our results would appear to be that involved in the assumption that the circle is perfectly rigid. Changes in the form of the circle due to change of temperature, &c., can, of course, only be guarded against by the process already proposed for the elimination of changes in the adjustment of the microscopes. There remain, however, to be considered the possible effects of a small degree of flexibility of the circle.

The only cause which could give rise to flexure of the Cape

circles in their own plane is the gravitational stress to which they are subject. The circles being supported by continuous discs, and not by spokes, unless there is some want of homogeneity in the construction of these discs we may anticipate that the displacement of all division marks from this cause when brought under a given microscope will be the same. Thus the errors of flexure will be inseparably involved with the errors of adjustment of the microscopes, and any process designed to eliminate the latter will also of necessity likewise eliminate the former.

Whatever be the construction of the circles we may at least assume that the strain due to gravitational stress will be reversed in sign with the stress which produces it, *i.e.* it will affect with opposite signs and by equal amounts the mean readings of a pair of opposite microscopes at two settings of the circle differing by 180° . Thus we may completely eliminate all errors due to a possible flexibility of the circle in its own plane by repeating all operations in positions of the circle differing by 180° .

The following "sets" of operations are suggested as being sufficiently short to allow of reasonable constancy in the instrumental conditions throughout a set. An observer can easily perform two or more such sets at a single sitting without undue strain on the eyesight.

I. The division x is brought in succession under each of the six primary microscopes, and all six microscopes are read at each setting.

The same series of operations is immediately repeated in reversed order.

Such a set involves seventy-two pointings.

II. The divisions x , $x+20^\circ$, $x+40^\circ$ are brought in succession first under one of the primary microscopes A, and secondly under the opposite microscope B, and at each setting the microscopes A, B and the pair of subsidiary microscopes α , β situated at an interval of 20° from a pair of primary microscopes are read.

The same operations are immediately repeated in reversed order.

Such a set involves 48 pointings.

III. The divisions x , $x+5^\circ$, $x+10^\circ$, $x+15^\circ$ are brought in succession first under a microscope A and secondly under the opposite microscope B, and at each setting the microscopes A, B and the second pair of subsidiary microscopes α , β , situated at an interval of 25° from a pair of primary microscopes, are read.

The same operations are repeated in reversed order. Such a set will involve 64 pointings.

IV. The divisions x , $x+1^\circ$; $x+1^\circ$, $x+2^\circ$; $x+2^\circ$, $x+3^\circ$; $x+3^\circ$, $x+4^\circ$; $x+4^\circ$, $x+5^\circ$ are brought in succession under the divided object glass microscope, and the micrometer is pointed at the first setting on the images of the division marks x , $x+1^\circ$; at the second on those of the division marks $x+1^\circ$, $x+2^\circ$, and so on; *i.e.* the succession intervals by which the division marks x , $x+1^\circ$,

$x + 2^\circ$, $x + 3^\circ$, $x + 4^\circ$, $x + 5^\circ$ are separated are compared in order with the fixed interval by which the two images of a single division mark are separated by the divided object glass.

The same operations are then repeated in reversed order.

Each set will involve 20 pointings.

§ 4. Determination of Six Fundamental Intervals.

Consider first a set of observations of Type I. We have in all 72 pointings, from which, if we combine into a single observation the means of the results obtained from the direct and reversed orders of measurement, we may derive 36 equations of condition.

Denote by x_o , x_{60} , x_{120} , x_{180} , x_{240} , x_{300} the division errors of the six division marks involved, and by y_A , y_B , y_C , y_D , y_E , y_F the errors of adjustment of the six microscopes; also let z_1 , z_2 , z_3 , z_4 , z_5 , z_6 denote the errors in setting in the six positions of the circle.

Then, disregarding the errors of the pivots, the 36 equations of condition may be written in the form

$$\left. \begin{aligned} x_o + y_A + z_1 &= m_{11}, & x_{60} + y_B + z_1 &= m_{12}, & x_{120} + y_C + z_1 &= m_{13}, \\ x_{180} + y_D + z_1 &= m_{14}, & x_{240} + y_E + z_1 &= m_{15}, & x_{300} + y_F + z_1 &= m_{16} \\ x_o + y_B + z_2 &= m_{21}, & x_{60} + y_C + z_2 &= m_{22}, & x_{120} + y_D + z_2 &= m_{23}, \\ x_{180} + y_E + z_2 &= m_{24}, & x_{240} + y_F + z_2 &= m_{25}, & x_{300} + y_A + z_2 &= m_{26} \\ x_o + y_C + z_3 &= m_{31}, & x_{60} + y_D + z_3 &= m_{32}, & x_{120} + y_E + z_3 &= m_{33}, \\ x_{180} + y_F + z_3 &= m_{34}, & x_{240} + y_A + z_3 &= m_{35}, & x_{300} + y_B + z_3 &= m_{36} \\ x_o + y_D + z_4 &= m_{41}, & x_{60} + y_E + z_4 &= m_{42}, & x_{120} + y_F + z_4 &= m_{43}, \\ x_{180} + y_A + z_4 &= m_{44}, & x_{240} + y_B + z_4 &= m_{45}, & x_{300} + y_C + z_4 &= m_{46} \\ x_o + y_E + z_5 &= m_{51}, & x_{60} + y_F + z_5 &= m_{52}, & x_{120} + y_A + z_5 &= m_{53}, \\ x_{180} + y_B + z_5 &= m_{54}, & x_{240} + y_C + z_5 &= m_{55}, & x_{300} + y_D + z_5 &= m_{56} \\ x_o + y_F + z_6 &= m_{61}, & x_{60} + y_A + z_6 &= m_{62}, & x_{120} + y_B + z_6 &= m_{63}, \\ x_{180} + y_C + z_6 &= m_{64}, & x_{240} + y_D + z_6 &= m_{65}, & x_{300} + y_E + z_6 &= m_{66} \end{aligned} \right\} \quad (A)$$

Now on taking the mean of all the equations which involve x_o , i.e. those which appear in the first column, we obtain

$$x_o + \bar{y} + \bar{z} = \frac{1}{6}(m_{11} + m_{21} + m_{31} + m_{41} + m_{51} + m_{61}) \quad (B)$$

where $\bar{y}$, $\bar{z}$ denote the means of all the quantities y , z involved.

By taking the mean of all the equations we derive

$$\bar{x} + \bar{y} + \bar{z} = \frac{1}{36} \Sigma m \quad \dots \quad \dots \quad \dots \quad (C)$$

Taking the difference of the two equations (B) and (C) we can completely eliminate the quantities y , z and derive

$$x_o - \bar{x} = \frac{1}{6}(m_{11} + m_{21} + m_{31} + m_{41} + m_{51} + m_{61}) - \frac{1}{36} \Sigma m$$

with similar expressions for x_{60} , x_{120} , &c.

By combining the equations for x_o , x_{180} we obtain

$$\frac{1}{2}(x_o + x_{180}) - \bar{x} = \frac{1}{12}(m_{11} + m_{21} + m_{31} + m_{41} + m_{51} + m_{61} + m_{14} + m_{24} + m_{34} + m_{44} + m_{54} + m_{64}) - \frac{1}{36}\Sigma m.$$

We are thus enabled to eliminate the quantities y , z , and to determine the errors of division of a division mark or of a pair of opposite division marks, but only in relation to the mean of all the six division marks under investigation. Denote the division error of the mark o° with relation to these six divisions by ξ_o , that of the pair of opposite division marks o° , 180° by X_o , so that

$$\xi_o = x_o - \bar{x} = \frac{1}{6}(m_{11} + m_{21} + m_{31} + m_{41} + m_{51} + m_{61}) - \frac{1}{36}\Sigma m,$$

$$X_o = \frac{1}{2}(x_o + x_{180}) - \bar{x} = \frac{1}{12}(m_{11} + m_{21} + m_{31} + m_{41} + m_{51} + m_{61} + m_{14} + m_{24} + m_{34} + m_{44} + m_{54} + m_{64}) - \frac{1}{36}\Sigma m.$$

Since each of the quantities m depends on the mean of two pointings, if we suppose that the mean error of a single pointing is ϵ , the square of the mean error of each m is $\frac{1}{2}\epsilon^2$.

In the expression for ξ_o all 36 m 's appear, 6 of them with a coefficient $\frac{5}{36}$ and the remaining 30 with a coefficient $-\frac{1}{36}$.

Hence the square of the mean accidental error of ξ_o is

$$\frac{1}{2}\epsilon^2[6(\frac{5}{36})^2 + 30(\frac{1}{36})^2] = \frac{5\epsilon^2}{72}.$$

In like manner the square of the mean accidental error of X_o is

$$\frac{1}{2}\epsilon^2[12(\frac{2}{36})^2 + 24(\frac{1}{36})^2] = \frac{1}{36}\epsilon^2.$$

Thus the weight of the determination of a single division error, assumed unaffected by systematic error, is $\frac{72}{5}$, that of a pair of opposite divisions is 36, unit weight corresponding with that of a single micrometer pointing.

The difference of two of the quantities X —say X_o , X_{60} —will be expressible in the form

$$X_o - X_{60} = \frac{1}{12}[m_{11} + m_{21} + m_{31} + m_{41} + m_{51} + m_{61} + m_{14} + m_{24} + m_{34} + m_{44} + m_{54} + m_{64} - m_{12} - m_{22} - m_{32} - m_{42} - m_{52} - m_{62} - m_{15} - m_{25} - m_{35} - m_{45} - m_{55} - m_{65}]$$

whence the square of the mean error of $X_o - X_{60}$ is

$$\frac{1}{144}\left[24\frac{\epsilon^2}{2}\right] = \frac{\epsilon^2}{12},$$

and this is the square of the mean amount by which the measurement of an angle which is approximately a multiple of 60° , but not of 180° , will be in error on account of imperfect determina-

tion of division errors, if the angle be measured by reading a pair of opposite microscopes on the division marks now under investigation.

An angle which is approximately a multiple of 180° will not be affected by division errors if determined by means of a pair of opposite microscopes. In like manner, if all six microscopes be read at any two settings of the circle which involve the six divisions under investigation the errors of division of the circle will be entirely eliminated from the angle through which the circle has been rotated between the two settings.

§ 5. *Derivation of Probable Error.*

It may readily be shown that the process of combination of the equation (A) which we have used is the exact equivalent of the "method of least squares."* We may thus derive from the residuals an estimate of the probable error of a single equation or that of a single micrometer reading, according to the usual rules laid down for the latter process, with, however, a slight modification.

We have already seen that the quantities x are not in themselves determinate from the equations (A), but only their differences from their mean value. In like manner the quantities y or z cannot be completely determined from equations (A). In fact, the normal equations derived from equations (A) will not be independent, and in order to obtain an unique solution from them it will be necessary to impose on the quantities x, y, z two linear relations which may to a large extent be arbitrarily chosen; e.g. we might suppose

$$x_0 = 0, \quad y_A = 0.$$

A more symmetrical pair of relations which will equally serve the purpose, and enable us to arrive at determinate residuals, is as follows:

$$\Sigma x = 0 \quad \Sigma y = 0.$$

It is clear that the imposition of these conditions is equivalent to reducing the effective number of unknown quantities which enter into the left-hand members of the normal equations by 2, and hence the mean error of a single pointing must be derived by the formula

$$\epsilon^2 = \frac{\Sigma pvv}{m-n+2},$$

where p denotes the weight of a single equation,
 m the number of observation equations,
 n the total number of quantities derived,
 and v a "residual."

* P. H. Cowell, *Monthly Notices*, lxi. p. 527.

In the case of a set such as that under consideration we have

$$p = 2, \quad m = 36, \quad n = 18,$$

for we determine in all six quantities x , six quantities y , and six quantities z . Thus

$$\epsilon^2 = \frac{1}{18} \Sigma vv.$$

If, however, we combine into a single observation equation the pairs resulting from simultaneous pointings at opposite extremities of a diameter, we have

$$p = 4, \quad m = 18, \quad n = 12,$$

since we only determine three means of pairs of values of x and y , and six values of z .

Thus in this case

$$\epsilon^2 = \frac{1}{2} \Sigma vv,$$

and the derived value of the probable error will be independent of errors of the pivots.

If further we combine into a single observation equation the means of the four observations taken on a pair of opposite division marks in positions of the circle differing by 180° , so that the residuals will be free from errors of flexure, we have

$$p = 8, \quad m = 9, \quad n = 9,$$

and therefore

$$\epsilon^2 = 4 \Sigma vv.$$

§ 6. *Numerical Illustration.*

The following illustration is selected at random from among those already completed in connection with the Cape circles.

The division marks under investigation are the marks 0° , 60° , 120° , 180° , 240° , 300° . At the first setting of the circle the division mark 30° is brought under the pointer, and the marks under investigation are then respectively under the microscopes F, E, D, C, B, A. Care is taken to set the circle with such accuracy as to ensure that all measurements should be referred to the same parts of the micrometer screws. The results of a set (means of readings in direct and reversed directions of the circle) may then be expressed in the following tabular form, in which only the fractional parts of a revolution of the micrometer screw-head are quoted. One revolution of the screw-head is equivalent to a minute of arc, and the heads are divided into 100 parts.

Table of Micrometer Readings.

Pointer Reading.	Micr. A	Micr. B	Micr. C	Micr. D	Micr. E	Micr. F
0	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>
30	·187	·142	·209	·270	·268	·196
90	·110	·104	·160	·218	·239	·185
150	·140	·106	·195	·242	·236	·191
210	·171	·130	·166	·243	·247	·188
270	·134	·102	·152	·199	·256	·192
330	·144	·116	·179	·223	·210	·193

The means of the quantities in the same horizontal line are respectively

$$\begin{array}{cccccc} \text{·212,} & \text{·169,} & \text{·185,} & \text{·191,} & \text{·172,} & \text{·177.} \end{array}$$

If we suppose that $\Sigma x = 0$, and $\Sigma y = 0$, these will represent the errors in setting, and on subtracting them from the separate entries we obtain the following revised table from which the errors in setting have been removed :—

Pointer reading.	Micr. A	Micr. B	Micr. C	Micr. D	Micr. E	Micr. F
0	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>
30	—·025	—·070	—·003	+·058	+·056	—·016
90	—·059	—·065	—·009	+·049	+·070	+·016
150	—·045	—·079	+·010	+·057	+·051	+·006
210	—·020	—·061	—·025	+·252	+·056	—·003
270	—·038	—·070	—·020	+·026	+·083	+·019
330	—·033	—·061	+·001	+·045	+·032	+·015
Means	—·037	—·068	—·008	+·048	+·058	+·006

The means here quoted represent the relative errors of adjustment of the micrometers. Subtracting them from the separate entries we derive the following table of division errors free from errors in setting of the circle and errors in adjustment of the micrometers :—

Pointer Reading.	Micr. A	Micr. B	Micr. C	Micr. D	Micr. E	Micr. F
0	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>
30	+·012	—·002	+·005	+·010	—·002	—·022
90	—·022	+·003	—·001	+·001	+·012	+·010
150	—·008	—·011	+·018	+·009	—·007	·000
210	+·017	+·007	—·017	+·004	—·002	—·009
270	—·002	—·003	—·013	—·022	+·025	+·013
330	+·003	+·006	+·009	—·002	—·026	+·009

The entries which correspond with a given division mark will appear in diagonals of this table. Collecting them into the same vertical columns, we obtain :

Division Mark	300°	240°	180°	120°	60°	0°
Pointer.	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>
30	+ '012	- '002	+ '005	+ '010	- '002	- '022
90	+ '003	- '001	+ '001	+ '012	+ '010	- '022
150	+ '018	+ '009	- '007	'000	- '008	- '011
210	+ '004	- '002	- '009	+ '017	+ '007	- '017
270	+ '025	+ '013	- '002	- '003	- '013	- '022
330	+ '009	+ '003	+ '006	- '002	- '002	- '026
Means	+ '0118	+ '0033	+ '0010	- '0013	- '0013	- '0200

The residuals from the means are as follows :

Division Mark	300°	240°	180°	120°	60°	0°
Pointer.	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>r</i>
30	+ '0002	- '0053	+ '0060	+ '0025	- '0007	- '0020
90	- '0088	- '0043	+ '0020	+ '0045	+ '0113	- '0020
150	+ '0062	+ '0057	- '0060	- '0075	- '0067	+ '0090
210	- '0078	- '0053	- '0080	+ '0095	+ '0083	+ '0030
270	+ '0132	+ '0097	- '0010	- '0105	- '0117	- '0020
330	- '0028	- '0003	+ '0070	+ '0015	- '0007	- '0060

and the sum of their squares expressed in units of the fourth decimal place is

$$156 \cdot 302 ;$$

whence the square of the mean error of a single pointing is

$$156 \cdot 30,$$

and the probable error of a single pointing $\pm 0^{\circ} \cdot 0083 = \pm 0'' \cdot 50$.

The probable accidental error of a single pointing, as evidenced by the interagreement of the pointings made in direct and reversed measurements, can scarcely amount to as large a quantity as this, and it would thus appear that there is some considerable source of systematic error affecting the interagreement of our results. To ascertain whether the discordances may be attributed to pivot errors we combine, by taking means, the pairs of residuals derived from opposite microscopes at the same setting of the circle. The residuals now become :

Division Mark			
Pointer.	300° and 120°	240° and 60°	0° and 0°
30	+ '0014	- '0030	+ '0020
90	- '0022	+ '0035	'0000
150	- '0006	- '0004	+ '0015
210	+ '0009	+ '0015	- '0025
270	+ '0014	- '0010	- '0015
330	- '0007	- '0005	+ '0005

The sum of the squares of the residuals expressed in units of the fourth decimal place now amounts to

$$5033,$$

and the square of the mean error of a single pointing as derived from the interagreement of the several determinations of the division errors of pairs of opposite division marks is

$$\frac{1}{2}\Sigma vv = 2517.$$

The probable accidental error of a single pointing as thus derived is therefore $\pm 0''.0033$ or $\pm 0''.20$.

This agreement is now satisfactory, the true value of the probable accidental error of a single pointing as derived by various tests involving a very large number of pointings being apparently about $\pm 0''.18$.

To ascertain whether the agreement might be still further improved by attributing a small amount of flexure to the circle, we combine into a single residual the means of the four resulting from pointings on two opposite divisions under each of two opposite microscopes.

The resulting residuals are given in the following table :

Division Marks			
Pointer Readings.	300° and 120°	240° and 60°	180° and 0°
30 and 210	+ '0012	- '0008	- '0003
90 " 270	- '0004	+ '0013	- '0008
150 " 330	- '0007	- '0005	+ '0010

The sum of the squares of these residuals in units of the fourth decimal place is

$$640,$$

and the square of the mean error of a single pointing

$$= 4vv = 2560.$$

Thus no flexure exists of sufficient amount to materially influence the agreement of the observations of the present series.

The same conclusion is borne out from a far more extended series of observations.

§ 7. Determination of 20° Spaces.

Consider next the significance of a set of observations of type II. Such a set consists of forty-eight pointings, but we can combine into a single equation the means of the pointings made in direct and reversed orders of measurement, and we shall obtain twenty-four equations of condition, which we may write as follows :

$$\begin{aligned} x_o + y_A + z_1 &= m_{11}, & x_{20} + y_a + z_1 &= m_{12}, & x_{180} + y_B + z_1 &= m_{13}, & x_{200} + y_b + z_1 &= m_{14} \\ x_{20} + y_A + z_2 &= m_{21}, & x_{40} + y_a + z_2 &= m_{22}, & x_{200} + y_B + z_2 &= m_{23}, & x_{220} + y_b + z_2 &= m_{24} \\ x_{40} + y_A + z_3 &= m_{31}, & x_{60} + y_a + z_3 &= m_{32}, & x_{220} + y_B + z_3 &= m_{33}, & x_{240} + y_b + z_3 &= m_{34} \\ x_{180} + y_A + z_4 &= m_{41}, & x_{200} + y_a + z_4 &= m_{42}, & x_o + y_B + z_4 &= m_{43}, & x_{20} + y_b + z_4 &= m_{44} \\ x_{200} + y_A + z_5 &= m_{51}, & x_{220} + y_a + z_5 &= m_{52}, & x_{20} + y_B + z_5 &= m_{53}, & x_{40} + y_b + z_5 &= m_{54} \\ x_{220} + y_A + z_6 &= m_{61}, & x_{240} - y_a + z_6 &= m_{62}, & x_{40} + y_B + z_6 &= m_{63}, & x_{60} + y_b + z_6 &= m_{64} \end{aligned}$$

If we write for brevity

$$\begin{aligned} X_o &= \frac{1}{2}(x_o + x_{180}), & X_{20} &= \frac{1}{2}(x_{20} + x_{200}), & \&c., \\ Y_I &= \frac{1}{2}(y_A + y_B) - \frac{1}{2}(y_a + y_b), \end{aligned}$$

by combining the equations which appear in the same horizontal line we may completely eliminate all the quantities z and all the quantities y except as they appear in the combination Y_I .

Thus we obtain

$$\begin{aligned} X_o - X_{20} + Y_I &= \frac{1}{2}(m_{11} + m_{13}) - \frac{1}{2}(m_{12} + m_{14}) \\ X_{20} - X_{40} + Y_I &= \frac{1}{2}(m_{21} + m_{23}) - \frac{1}{2}(m_{22} + m_{24}) \\ X_{40} - X_{60} + Y_I &= \frac{1}{2}(m_{31} + m_{33}) - \frac{1}{2}(m_{32} + m_{34}) \\ X_o - X_{20} + Y_I &= \frac{1}{2}(m_{41} + m_{43}) - \frac{1}{2}(m_{42} + m_{44}) \\ X_{20} - X_{40} + Y_I &= \frac{1}{2}(m_{51} + m_{53}) - \frac{1}{2}(m_{52} + m_{54}) \\ X_{40} - X_{60} + Y_I &= \frac{1}{2}(m_{61} + m_{63}) - \frac{1}{2}(m_{62} + m_{64}). \end{aligned}$$

Since each m has weight 2, the weight of each absolute term of these equations will be 2. Taking the means of the first and fourth equations, the second and fifth, and the third and sixth, we derive three equations of the type

$$\begin{aligned} X_o - X_{20} + Y_I &= n_1 \\ X_{20} - X_{40} + Y_I &= n_2 \\ X_{40} - X_{60} + Y_I &= n_3, \end{aligned}$$

in which each absolute term will be of weight 4.

As these equations involve five unknown quantities, X_o , X_{2o} , X_{4o} , X_{6o} , Y_1 , they will not lead to a determination of any of the unknowns except in combination with other observations. If, however, we suppose X_o , X_{6o} as already known from operations of Type I., we may derive X_{2o} , X_{4o} , Y_1 without ambiguity. We have, in fact,

$$\begin{aligned} Y_1 &= \frac{1}{3}(n_1 + n_2 + n_3) - \frac{1}{3}(X_o - X_{6o}) \\ X_{2o} &= \frac{1}{3}(n_2 + n_3 - 2n_1) + \frac{1}{3}(2X_o + X_{6o}) \\ X_{4o} &= \frac{1}{3}(2n_3 - n_1 - n_2) + \frac{1}{3}(X_o + 2X_{6o}). \end{aligned}$$

The quantities n_1 , n_2 , n_3 each being of weight 4, if X_o , X_{6o} be supposed absolutely known, the square of the mean errors of X_{2o} , X_{4o} will be

$$\frac{1}{4}\epsilon^2\left(\frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{1}{6}\epsilon^2.$$

If, however, X_o , X_{6o} have only been fallibly determined by means of sets of observations of Type I., concentrated on the set of division marks 0° , 60° , 120° , 180° , 240° , 300° , we may suppose that they have been subjected to the condition

$$x_o + x_{6o} + x_{12o} + \dots = 0$$

or

$$X_o + X_{6o} + X_{12o} = 0;$$

we shall then have

$$\begin{aligned} 2X_o + X_{6o} &= X_o - X_{12o} \\ X_o + 2X_{6o} &= X_{6o} - X_{12o} \end{aligned}$$

and each of these quantities has been shown to be determined with weight 12 by means of a single set of Type I. Thus the square of the mean error of a single determination of

$$\frac{1}{3}(2X_o + X_{6o}) \text{ or } \frac{1}{3}(X_o + 2X_{6o})$$

will be

$$\frac{1}{108}\epsilon^2.$$

Hence the square of the mean error of X_{2o} , X_{4o} supposed to depend on p sets of operations of Type I. and q sets of Type II. will be

$$\left(\frac{1}{108p} + \frac{1}{6q}\right)\epsilon^2.$$

It is evident from this formula that the operations of Type II. will have to be very frequently repeated if it is desired to determine the quantities X_{2o} , X_{4o} with anything approaching the precision with which the errors of the fundamental diameters are determined even from a single set of operations of Type I.

Since operations of Type I. are so much more effective than those of Type II., the question arises as to whether it would not be advantageous to subject the intermediate division marks x_{2o} , x_{4o} , &c., directly to sets of operations of the first type.

§ 8. *Alternative Method of Combination of Observations.*

Suppose that three sets of observations of Type II. have been made, subdividing the intervals between the three fundamental diameters 0° , 60° , 120° . Let the resulting equations be expressed in the form

$$\begin{aligned} X_0 - X_{20} + Y_1 &= n_1, & X_{60} - X_{80} + Y_2 &= n_4, & X_{120} - X_{140} + Y_3 &= n_7 \\ X_{20} - X_{40} + Y_1 &= n_2, & X_{80} - X_{100} + Y_2 &= n_5, & X_{140} - X_{160} + Y_3 &= n_8 \\ X_{40} - X_{60} + Y_1 &= n_3, & X_{100} - X_{120} + Y_2 &= n_6, & X_{160} - X_{180} + Y_3 &= n_9 \end{aligned}$$

Then if we put

$$\begin{aligned} \frac{1}{3}(X_0 + X_{60} + X_{120}) &= A_0, \\ \frac{1}{3}(X_{20} + X_{80} + X_{140}) &= A_{20}, \\ \frac{1}{3}(X_{40} + X_{100} + X_{160}) &= A_{40}, \end{aligned}$$

on taking means of the equations in the same horizontal line we obtain

$$\begin{aligned} A_0 - A_{20} + [Y] &= \frac{1}{3}(n_1 + n_4 + n_7) = N_1 \\ A_{20} - A_{40} + [Y] &= \frac{1}{3}(n_2 + n_5 + n_8) = N_2 \\ A_{40} - A_0 + [Y] &= \frac{1}{3}(n_3 + n_6 + n_9) = N_3, \end{aligned}$$

where

$$[Y] = \frac{1}{3}(Y_1 + Y_2 + Y_3).$$

Since, if the observations depend only on a single set, each n will have weight 4, each N will have weight 12. As, however, these equations involve four unknown quantities, A_0 , A_{20} , A_{40} , $[Y]$, they cannot be solved without being supplemented with some further condition. If we suppose that

$$A_0 + A_{20} + A_{40} = 0,$$

i.e. that the sum of the division errors of all the 18 division marks now under investigation is zero, we derive

$$\begin{aligned} A_0 &= \frac{1}{3}(N_1 - N_3), & A_{20} &= \frac{1}{3}(N_2 - N_1), & A_{40} &= \frac{1}{3}(N_3 - N_2), \\ [Y] &= \frac{1}{3}(N_1 + N_2 + N_3), \end{aligned}$$

Since each N has weight 12, the square of the mean error of each A will be

$$\frac{2}{9} \left[\frac{\epsilon^2}{12} \right] = \frac{1}{54} \epsilon^2$$

and the weight of each of the A 's will be 54.

Again, we have

$$A_0 - A_{20} = \frac{1}{3}(2N_1 - N_2 - N_3),$$

whence the square of the mean error of $A_0 - A_{20}$ is

$$\frac{\epsilon^2}{12} \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) = \frac{\epsilon^2}{18},$$

and its weight is 18.

Now if each of the quantities $X_o, X_{20}, X_{40},$ &c., have been previously derived from operations of Type I. subject to the conditions

$$X_o + X_{60} + X_{120} = 0$$

$$X_{20} + X_{80} + X_{140} = 0$$

$$X_{40} + X_{100} + X_{160} = 0,$$

the quantities A_o, A_{20}, A_{40} will represent the corrections applicable to the different groups of division errors as thus derived, in order to refer the whole series of division marks to a system defined by

$$X_o + X_{20} + X_{40} + X_{60} + X_{80} + X_{100} + X_{120} + X_{140} + X_{160} = 0.$$

Since the quantities X_o are derived from a single set of operations of Type I. with weight 36, and the quantities A_o from a single series of sets of Type II. with weight 54, the square of the mean error of the resulting division error with reference to the set of 18 division marks is

$$\epsilon^2 \left(\frac{1}{36} + \frac{1}{54} \right) = \frac{5\epsilon^2}{108}.$$

Thus the weight of the determination of each division error will be $21\frac{2}{3}$, and the repetition of the operations of Type I. on different series of fundamental divisions has increased the weight of the results more than threefold.

To determine the most economical distribution of the observations in order to derive the relative division errors of a set of 18 division marks with the highest precision, consider the following data :—

If the operations of Type I. be repeated p times and those of Type II. q times, the weight of the resulting determinations will be

$$\frac{1}{\frac{1}{36p} + \frac{1}{54q}} = \frac{108pq}{2p + 3q}.$$

Each set of Type I. will involve 72 pointings, the series of three sets will involve 216 pointings, and if repeated p times 216 p pointings will be required.

Each set of Type II. will involve 48 pointings and a series of three sets each repeated q times will involve 144 q pointings.

Hence the best distribution of the observations is that which with a given value of

$$216p + 144q [= 72(3p + 2q)]$$

gives the greatest value to

$$\frac{108pq}{2p + 3q}.$$

Now

$$\frac{pq}{2p+3q} = (3p+2q) \frac{pq}{6p^2+6q^2+13pq}$$

$$= (3p+2q) \frac{1}{\left(\sqrt{\frac{p}{q}} - \sqrt{\frac{q}{p}}\right)^2 + 25}.$$

Subject to a given value of $3p+2q$ this will evidently be greatest when

$$\sqrt{\frac{p}{q}} - \sqrt{\frac{q}{p}} = 0,$$

i.e. when

$$p = q.$$

Hence it is desirable that any repetition of the observations should as far as possible be symmetrically distributed over the sets of Type I. and Type II.

§ 9. Determination of 5° Spaces.

The subdivision of the 20° spaces into spaces of 5° interval may be effected by means of series of operations of Type III. Exactly as in the case of the last series, a set of observations may be supposed to lead to a set of equations of condition of the type

$$X_0 - X_{25} + Y_1 = n_1$$

$$X_5 - X_{30} + Y_1 = n_2$$

$$X_{10} - X_{35} + Y_1 = n_3$$

$$X_{15} - X_{40} + Y_1 = n_4,$$

in which each absolute term is of weight 4.

By means of nine such sets, starting at intervals of 20° round the circle, we can obtain thirty-six equations of the type

$$X_0 - X_{25} + Y_1 = n_1, \quad X_5 - X_{30} + Y_1 = n_2, \quad X_{10} - X_{35} + Y_1 = n_3, \quad X_{15} - X_{40} + Y_1 = n_4,$$

$$X_{20} - X_{45} + Y_2 = n_5, \quad X_{25} - X_{50} + Y_2 = n_6, \quad X_{30} - X_{55} + Y_2 = n_7, \quad X_{35} - X_{60} + Y_2 = n_8,$$

$$X_{40} - X_{65} + Y_3 = n_9, \quad X_{45} - X_{70} + Y_3 = n_{10}, \quad X_{50} - X_{75} + Y_3 = n_{11}, \quad X_{55} - X_{80} + Y_3 = n_{12}$$

&c.,

&c.,

where the equations resulting from a single set are now written in the same horizontal line.

If we suppose that

$$\frac{1}{9}(X_0 + X_{20} + X_{40} + X_{60} + X_{80} + X_{100} + X_{120} + X_{140} + X_{160}) = B_0,$$

$$\frac{1}{9}(X_5 + X_{25} + X_{45} + \dots \dots \dots + X_{165}) = B_5,$$

$$\frac{1}{9}(X_{10} + X_{30} + X_{50} + \dots \dots \dots + X_{170}) = B_{10},$$

$$\frac{1}{9}(X_{15} + X_{35} + X_{55} + \dots \dots \dots + X_{175}) = B_{15},$$

on taking the means of all the equations in the same vertical column, we derive

$$B_0 - B_5 + [Y] = N_1$$

$$B_5 - B_{10} + [Y] = N_2$$

$$B_{10} - B_{15} + [Y] = N_3$$

$$B_{15} - B_0 + [Y] = N_4$$

where

$$[Y] = \frac{1}{9}(Y_1 + Y_2 + Y_3 + \dots + Y_9)$$

and each absolute term is of weight 36.

These equations must be supplemented by a further condition to render B_0 , B_5 , &c., determinate. Suppose we assume that

$$B_0 + B_5 + B_{10} + B_{15} = 0$$

i.e. that the sum of division errors of all seventy-two division marks now involved is zero. We then find

$$[Y] = \frac{1}{4}(N_1 + N_2 + N_3 + N_4)$$

$$B_0 = \frac{1}{8}(3N_1 + N_2 - N_3 - 3N_4)$$

$$B_5 = \frac{1}{8}(3N_2 + N_3 - N_4 - 3N_1)$$

$$B_{10} = \frac{1}{8}(3N_3 + N_4 - N_1 - 3N_2)$$

$$B_{15} = \frac{1}{8}(3N_4 + N_1 - N_2 - 3N_3)$$

Since each N has weight 36, the square of the mean error of each B will be

$$\frac{\epsilon^2}{36} \cdot \frac{9+1+1+9}{64} = \frac{5\epsilon^2}{576}$$

The difference of two of the quantities B , say B_0 and B_5 , is

$$\frac{1}{4}(3N_1 - N_2 - N_3 - N_4)$$

and the square of its mean error is

$$\frac{\epsilon^2}{36} \cdot \frac{9+1+1+1}{16} = \frac{1}{48} \epsilon^2$$

To utilise these observations in combination with previous sets we may suppose that by means of the former sets we have derived values of

$$X_0, X_{20}, X_{40}, \dots, X_{160}$$

but only on the assumption that

$$A_0 + A_{20} + A_{40} = 0$$

or, what is equivalent,

$$X_0 + X_{20} + X_{40} + \dots + X_{160} = 0$$

If the true value of this quantity be B_0 , each X of the series as

previously derived will require a correction B_0 . Suppose similar sets have been made to connect the division marks

$$\begin{array}{l} X_5, X_{25}, X_{45}, \dots \\ X_{10}, X_{30}, X_{40}, \dots \\ X_{15}, X_{35}, X_{55}, \dots \end{array}$$

The derived values of the X 's must be corrected by the appropriate amounts B common to each series as derived from observations of the present type.

Thus each division error will consist of three parts :

(1) A part derivable from a series of operations of Type I., which expresses the error of the division mark relatively to six symmetrically situated division marks of which it is one.

(2) A correction A , derived from observations of Type II., to refer it to a set of eighteen symmetrically situated divisions of which it is one.

(3) A correction B , derived from observations of Type III., to refer it to the set of seventy-two symmetrically situated divisions in which occurs.

Since these parts are independently derived, the square of the mean error of the result as thus obtained will be the sum of the squares of the mean errors of its component parts. Thus, if each division error is determined as the result of only one complete series of operations of each type, the square of its mean error will be

$$\epsilon^2 \left(\frac{1}{36} + \frac{1}{54} + \frac{5}{576} \right)$$

If the sets of observations of Type I. be repeated p times, those of Type II. q times, and those of Type III. r times, this expression will be replaced by

$$\epsilon^2 \left(\frac{1}{36p} + \frac{1}{54q} + \frac{5}{576r} \right) = \frac{\epsilon^2}{1728} \left(\frac{48}{p} + \frac{32}{q} + \frac{15}{r} \right)$$

and such a series of observations will involve

$$864p + 576q + 576r, \text{ or } 288(3p + 2q + 2r) \text{ pointings.}$$

Hence, in order to secure the highest precision with a given number of pointings, we must ascertain what values of p, q, r will make

$$\frac{48}{p} + \frac{32}{q} + \frac{15}{r}$$

a minimum, subject to the condition that

$$3p + 2q + 2r$$

should be constant.

By the ordinary rules of the calculus we find, on introducing an undetermined multiplier λ ,

$$\frac{48}{p^2} - 3\lambda = 0, \quad \frac{32}{q^2} - 2\lambda = 0, \quad \frac{15}{r^2} - 2\lambda = 0$$

Whence

$$\frac{p^2}{32} = \frac{q^2}{32} = \frac{r^2}{15}$$

and

$$\frac{p}{r} = \sqrt{\frac{32}{15}} = 1.46$$

or roughly

$$p : q : r = 3 : 3 : 2$$

This indicates how the work may best be distributed in order to secure the highest precision with the greatest economy of labour for the determination for the relative errors of a set of seventy-two equi-spaced division marks.

§ 10. *Determination of Single-degree Spaces.*

The operations previously discussed are restricted to a set of divisions symmetrically situated at intervals of 5° throughout the circle. If we suppose that these operations have been performed on each set of 5° marks, commencing with 0° , 1° , 2° , 3° , 4° , then exactly as we have combined operations on three sets of division marks spaced at intervals of 60° to determine the distribution of eighteen symmetrically situated division marks, and subsequently of four sets spaced at intervals of 20° to determine the distribution of seventy-two division marks, so we may combine the five series of operations on seventy-two division marks to obtain the distribution of the 360° division marks which correspond with the exact degrees on the circle by means of observations taken with the double object-glass micrometer.

A set of observations will lead to equations of condition of the type

$$\begin{aligned} x_0 - x_1 + Y &= n_1 \\ x_1 - x_2 + Y &= n_2 \\ x_2 - x_3 + Y &= n_3 \\ x_3 - x_4 + Y &= n_4 \\ x_4 - x_5 + Y &= n_5 \end{aligned}$$

where each absolute term is of weight unity, since it depends on the difference of two pointings each repeated twice in direct and reversed order of operations.

Seventy-two such sets will be required to complete the round of the circle, and if we put

$$\begin{aligned} \frac{1}{72}(x_0 + x_5 + x_{10} + \dots + x_{355}) &= C_0 \\ \frac{1}{72}(x_1 + x_6 + x_{11} + \dots + x_{356}) &= C_1 \text{ \&c.} \end{aligned}$$

on taking the means of the first, second, third, fourth, and fifth equations resulting from each set, we obtain

$$C_0 - C_1 + [Y] = N_1$$

$$C_1 - C_2 + [Y] = N_2$$

$$C_2 - C_3 + [Y] = N_3$$

$$C_3 - C_4 + [Y] = N_4$$

$$C_4 - C_0 + [Y] = N_5,$$

which, if supplemented with the condition

$$C_0 + C_1 + C_2 + C_3 + C_4 = 0$$

(i.e. if we suppose that the zero of division error is referred to the mean of the 360 division marks), lead to

$$[Y] = \frac{1}{5}(N_1 + N_2 + N_3 + N_4 + N_5)$$

$$C_0 = \frac{1}{5}(2N_1 + N_2 - N_4 - 2N_5)$$

$$C_1 = \frac{1}{5}(2N_2 + N_3 - N_5 - 2N_1)$$

$$C_2 = \frac{1}{5}(2N_3 + N_4 - N_1 - 2N_2)$$

$$C_3 = \frac{1}{5}(2N_4 - N_5 - N_2 - 2N_3)$$

$$C_4 = \frac{1}{5}(2N_5 + N_1 - N_3 - 2N_4)$$

The difference of two of the quantities C will be expressed by formulæ of the nature

$$C_0 - C_1 = \frac{1}{5}(4N_1 - N_2 - N_3 - N_4 - N_5)$$

Hence, since each N is derived as the mean of seventy-two quantities of unit weight, the square of the mean error of each of the C 's will be

$$\frac{\epsilon^2}{72} \left(\frac{4+1+1+1+4}{25} \right) = \frac{\epsilon^2}{180}$$

and that of the difference of two of them will be

$$\frac{\epsilon^2}{72} \left(\frac{16+1+1+1+1}{25} \right) = \frac{\epsilon^2}{90}$$

Thus the square of the mean error of the division errors of a pair of opposite division marks, supposed based on the minimum number of sets of the four types under investigation, which will lead to a complete determination of the 180 diameters involved will be

$$\epsilon^2 \left(\frac{1}{36} + \frac{1}{54} + \frac{5}{576} + \frac{1}{180} \right) = \frac{523}{8640} \epsilon^2 = \frac{\epsilon^2}{16.5}$$

i.e. each pair of division errors will be determined with weight 16.5.

The mean division error of a set of six symmetrically situated

division marks will be determined by operations of the Types II., III., IV., and thus we may omit the part of the error resulting from operations of Type I. The square of the mean error for a set of six divisions which appear simultaneously under the six primary microscopes will therefore be

$$\epsilon^2\left(\frac{1}{34} + \frac{5}{576} + \frac{1}{180}\right) = \frac{283}{8640}\epsilon^2 = \frac{\epsilon^2}{30} \text{ nearly.}$$

To summarise the results now obtained.

If a series of operations consisting of sixty sets of Type I., sixty of Type II., forty-five of Type III., and seventy-two of Type IV., involving in all 11,520 pointings, be made, we may derive the errors of division of pairs of opposite division marks in the form

$$X_o + A_o + B_o + C_o; X_i + A_i + B_i + C_i \text{ \&c.}$$

where

X denotes the part derived from operations of Type I. and has weight 36;

A denotes the part derived from operations of Type II. with weight 54;

B denotes the part derived from operations of Type III. with weight $\frac{576}{5}$;

C denotes the part derived from operations of Type IV. with weight 180.

Further, we have seen that the difference of two of the quantities X, A, B, C which result from simultaneous determinations will have weights 12, 18, 48, 90 respectively.

Now the values of X, A, B, which refer to two diameters separated by an angle which is an exact multiple of 1° , but not an exact multiple of 5° , will be independently derived, whereas the values of C involved will have resulted from simultaneous determination. Hence the square of the mean error affecting the angle will be

$$\epsilon^2\left(\frac{2}{36} + \frac{2}{54} + \frac{5}{288} + \frac{1}{90}\right) = \frac{\epsilon^2}{8.6}$$

If the diameters be inclined at an angle which is a multiple of 5° , but not a multiple of 20° , the square of the mean error becomes

$$\epsilon^2\left(\frac{2}{36} + \frac{2}{54} + \frac{1}{48}\right) = \frac{\epsilon^2}{8.8}$$

since the quantities C involved will be the same for both diameters, and therefore will not enter into the angle expressed by their difference, while the B's will have been derived by a simultaneous determination.

In like manner the angle between two diameters, if a multiple of 20° but not a multiple of 60° , will be determined with an error whose mean square is

$$\epsilon^2\left(\frac{2}{36} + \frac{1}{18}\right) = \frac{\epsilon^2}{9}$$

since the quantities B, C will be the same for both diameters, while the A's will have been simultaneously determined.

Finally the angle between two diameters separated by 60° or any multiple thereof will be determined with weight 12, and the square of its mean error will be

$$\frac{\epsilon^2}{12}$$

These errors represent the amounts by which the measurement of an angle will be affected by imperfect determination of division error if the measurements are made with a pair of opposite microscopes. If three such pairs be used we may omit the parts of the errors resulting from imperfect determination of X, since the X's have been so determined that their mean value for six symmetrically placed division marks is zero.

Thus an angle which is a multiple of 1° , but not a multiple of 5° , will be affected by an amount whose mean square is

$$\epsilon^2\left(\frac{2}{34} + \frac{5}{288} + \frac{1}{90}\right) = \frac{\epsilon^2}{15} \text{ nearly ;}$$

an angle which is a multiple of 5° , but not a multiple of 20° , will be affected by an amount whose mean square is

$$\epsilon^2\left(\frac{2}{34} + \frac{1}{48}\right) = \frac{\epsilon^2}{17} \text{ nearly ;}$$

an angle which is a multiple of 20° , but not a multiple of 60° , will be affected by an amount whose mean square is

$$\frac{\epsilon^2}{18} ;$$

while an angle of 60° , or any multiple thereof, will be entirely free from the effects of division errors.

If the probable error of a single pointing be $\pm 0''.18$, which appears to about represent the precision attainable with the Cape micrometers, in the most unfavourable case the probable outstanding effect of division error on the determination of an angle measured with the aid of six microscopes will be

$$\pm 0''.047$$

§ 11. *Alternative Method of Reduction.*

The process of subdivision above described might be continued with suitable appliances until the relative division errors of every division mark on the circle had been ascertained. As, however, the circles are divided into arcs of 5' this will involve the determination of no fewer than 4320 unknown quantities, and the labour might well be regarded as prohibitive. We proceed to consider how it may best be further reduced.

Suppose, in the first instance, that the operations of Types I., II., III. had been restricted to the set of division marks 0° , 5° , 10° , . . . Then each set of Type IV. will, as before, lead to a set of equations of the form

$$x_0 - x_1 + Y = n_1$$

$$x_1 - x_2 + Y = n_2$$

$$x_2 - x_3 + Y = n_3$$

$$x_3 - x_4 + Y = n_4$$

$$x_4 - x_5 + Y = n_5$$

from which we may obtain

$$\frac{1}{5}(x_0 - x_5) + Y = \frac{1}{5}(n_1 + n_2 + n_3 + n_4 + n_5)$$

and

$$x_1 = \frac{4x_0 + x_5}{5} + \frac{1}{5}(-4n_1 + n_2 + n_3 + n_4 + n_5)$$

$$x_2 = \frac{3x_0 + 2x_5}{5} + \frac{1}{5}(-3n_1 - 3n_2 + 2n_3 + 2n_4 + 2n_5)$$

$$x_3 = \frac{2x_0 + 3x_5}{5} + \frac{1}{5}(-2n_1 - 2n_2 - 2n_3 + 3n_4 + 3n_5)$$

$$x_4 = \frac{x_0 + 4x_5}{5} + \frac{1}{5}(-n_1 - n_2 - n_3 - n_4 + 4n_5)$$

On combining the results of two sets taken at opposite extremities of diameters * we obtain

$$X_1 = \frac{4X_0 + X_5}{5} + \frac{1}{5}(-4N_1 + N_2 + N_3 + N_4 + N_5) \text{ \&c.}$$

where each N is the mean of two of the quantities n and therefore has weight 2.

If then we regard X_0 , X_5 as determined from previous operations, the square of the mean accidental error of X_1 , X_4

* The labour of observation will be somewhat reduced, while the errors of the pivots will be more effectively eliminated, if these two sets be taken simultaneously with a pair of microscopes provided with divided object-glasses. A second microscope of this character has accordingly been ordered from Messrs. Troughton and Simms.

resulting from the errors of observation in the present series will be

$$\frac{1}{2}\epsilon^2 \left(\frac{16+1+1+1+1}{25} \right) = \frac{2\epsilon^2}{5}$$

and that of X_2, X_3 will be

$$\frac{1}{2}\epsilon^2 \left(\frac{9+9+4+4+4}{25} \right) = \frac{3\epsilon^2}{5}$$

Thus in order to derive the quantities X_2, X_3 with the same precision as they may be derived by the previous process, it will be necessary to repeat the series of operations of Type IV. at least nine times, and this apart from the outstanding sources of error due to the previous imperfect determination of the quantities X_0, X_5 . The complete series of observations will involve no less than

$$72 \times 20 \times 9 (= 12960)$$

pointings, or about half as many again as would be required to repeat the operations of Types I., II., III. on different sets of division marks starting at $1^\circ, 2^\circ, 3^\circ, 4^\circ$.

Thus so long as the circle as a whole is under investigation it appears that it is highly advantageous to distribute as widely as possible the early observations; but the process now under discussion will be available for the more accurate determination of special divisions, *e.g.* the division marks called into play in the determinations of nadir readings, and the positions of close circumpolar stars may be regarded as worthy of special attention on account of the greater frequency with which they are put into subsequent use.

It has been shown that the complete set of operations requisite to determine the relative spacing of 360 division marks by means of sets of Type I., II., III., IV. will involve sixty sets of Type I., sixty of Type II., forty-five of Type III., and seventy-two of Type IV. This will require

$$60 \times 72 + 60 \times 48 + 45 \times 64 + 72 \times 20 (= 11520)$$

pointings.

In order to escape the difficulties resulting from irregularities of the pivots we have, however, confined our attention in the course of the discussion of such a set to the means of the errors of pairs of opposite division marks, which will be unaffected by pivot errors. We might have gone still further and restricted the discussion only to such errors as will affect the mean reading of the set of six primary microscopes. For any error which only affects the six readings of the microscopes differentially without affecting their mean will be automatically eliminated in the further use of the instrument, provided that all determinations are made with the aid of the six primary microscopes. Thus we

may omit all the operations of Type I. and the necessary number of pointings may be reduced by 4320.

Further, if the circles be free from flexure, the operations of Types II. and III. each involve a duplicate determination of the quantities A, B, since each division mark is brought under a pair of opposite microscopes, whereas the desired end could be attained without this reversal. This duplication of the observations, besides leading to the elimination of any flexure effects from the determination of the division errors, furnishes a valuable control of the observations, in addition to the primary control furnished throughout by the interagreement of the pointings in direct and reversed direction of motion of the circle, and it seems therefore undesirable to dispense with it ; if, however, we did so, the number of pointings necessary might be still further reduced by 2880.

There will remain 4320 pointings as the minimum number necessary for the complete investigation of the single-degree spaces.

§ 12. *General Conclusions.*

The process in general use for the determination of the division errors of a graduated circle consists in the determination of the intervals into which the complete circle is divided by a set of fundamental division marks, and of the successive subdivision of these intervals by subsidiary appliances, such as those attached to the Cape Transit circle, which are designed to admit of the subdivision of the circle to the extent of a single degree.

It is usual to concentrate a large amount of labour on the determination of the fundamental intervals, but the precision attained, at least so far as it depends on the repetition of observations, is necessarily reduced from step to step as the number of spaces to be divided increases. The precision finally obtained will, however, be limited by that of the final step, and it is shown that the labour involved in obtaining results from such a step, with precision at all commensurate with that which may be obtained by the present process from the sets which precede it, may always be more economically bestowed on the sets which precede.

The essential feature of the process now indicated consists in the distribution as widely as possible of the early operations rather than of their concentration on a set of particularly selected division marks. Apart from the high accuracy resulting therefrom, the method is characterised by extreme simplicity in the reductions and uniformity in the weights of the results.

Royal Observatory, Cape of Good Hope :
1903 December 22.

Note on the Date of the Passage of the Vernal Equinox from Taurus into Aries. By E. Walter Maunder and A. S. D. Maunder.

In vol. iii. of the *Transactions of the Society of Biblical Archaeology*, p. 237, Professor Sayce, after giving translations of a number of tablets, writes :—

“The reports quoted above which refer to the vernal equinox establish the fact that that period of the year corresponded with *Aries*. The Accadian calendar was arranged so as to suit the order of the zodiacal signs ; and Nisan the first month answered to the first zodiacal sign. Now the Sun still entered the first point of *Aries* at the vernal equinox in the time of Hipparchus, and would have done so since 2540 B.C. From that epoch backwards to 4698 B.C. *Taurus*, the second sign of the Accadian zodiac and the second month of the Accadian year, would have introduced the spring. The precession of the equinoxes thus enables us to fix the extreme limit of the antiquity of the ancient Babylonian calendar.”

The paper from which this quotation is made was written in 1874, and the course of Assyriological discovery has modified some of the conclusions which Professor Sayce then drew. But the two dates given were always evidently erroneous, and yet they are even to this day quoted as if they were of undoubted authority (e.g. R. Brown's *Primitive Constellations*, vol. i. p. 54 ; and E. M. Plunket's *Ancient Calendars and Constellations*, preface, p. viii). It seems worth while therefore to examine into the epochs when the vernal equinox lay within the several constellations, and the connection, if any, between the constellations of the zodiac and the months of the year.

The first point is to ascertain, as far as possible, what were the limits of the primitive zodiacal constellations. The oldest record of use in this respect is Ptolemy's Catalogue, the poem of Aratus giving but little help. It must be borne in mind that the form of that part of a constellation which is at a distance from the ecliptic is here of no interest. Thus, though the figure of *Aquarius* sprawls over nearly the entire length of *Capricornus* and for a good way under *Pisces*, the Sun can only have been considered as in *Aquarius* whilst it was passing along that portion of the ecliptic which lay wholly within the constellation. In the following table the latitudes and longitudes are given of a number of bright stars near the ecliptic and near the boundaries of their respective constellations, stars which there is every reason to believe still remain attached to the same figures as those to which they were originally assigned. From this table the second table has been prepared, giving the approximate longitude of the beginning of each zodiacal constellation, and of

the dates when the spring equinox coincided with it in the past, or will do so in the future. In the first table several extra-zodiacal stars have been included either on account of their brightness or as having been polar stars at some epoch.

TABLE I.

Table of Latitudes and Longitudes for 1900.

Star.			Magnitude.	Latitude.	Longitude.
γ Pegasi ...	...	...	2.87	+ 12 36	7 46
α Piscium ...	...	...	3.94	- 9 4	27 59
γ Arietis ...	...	...	4.04	+ 7 10	31 47
α Arietis ...	...	...	2.23	+ 9 57	36 15
δ Arietis ...	...	...	4.53	+ 1 49	49 27
η Tauri ...	...	...	2.96	+ 4 3	58 35
Aldebaran ...	...	...	1.06	- 5 32	68 23
Rigel ...	...	...	0.34	- 31 8	75 25
Capella ...	...	...	0.21	+ 22 52	80 27
β Tauri ...	...	...	1.78	+ 5 22	81 11
ζ Tauri ...	...	...	3.00	- 2 12	83 24
Polaris ...	...	...	2.12	+ 66 5	87 10
Betelgeux ...	...	...	var.	- 16 3	87 22
η Geminorum ...	...	...	1.93	- 0 54	92 2
Sirius ...	...	...	- 1.58	- 39 35	102 41
Castor ...	...	...	1.58	+ 10 5	108 51
Pollux ...	...	...	1.21	+ 6 40	111 50
Procyon ...	...	...	0.48	- 16 0	114 25
ζ Cancri ...	...	...	4.71	- 2 16	119 57
β Ursæ Minoris ...	...	...	2.24	+ 72 59	131 54
α Cancri ...	...	...	4.27	- 5 5	132 14
λ Leonis ...	...	...	4.48	+ 7 54	135 50
α Hydræ ...	...	...	2.16	- 22 26	145 47
Regulus ...	...	...	1.34	+ 0 27	148 26
α Draconis ...	...	...	3.64	+ 66 21	156 1
β Leonis ...	...	...	2.23	+ 12 17	170 14
β Virginis ...	...	...	3.80	+ 0 42	175 45
ϵ Virginis ...	...	...	2.95	+ 16 13	188 33
Spica ...	...	...	1.21	- 2 3	202 26
Arcturus ...	...	...	0.24	+ 30 48	202 50
λ Virginis ...	...	...	4.60	+ 0 32	215 34
α Libræ ...	...	...	2.90	+ 0 16	223 41
θ Libræ ...	...	...	4.34	+ 3 28	238 28

Star.			Magnitude.	Latitude.	Longitude.
♂ Scorpii ...	...	...	2.54	- 1 58	241 10
Antares ...	...	...	1.22	- 4 34	248 22
α Herculis ...	...	...	var.	+ 37 18	254 45
α Ophiuchi ...	...	...	2.14	+ 35 52	261 3
ε Scorpii ...	...	...	3.14	- 16 44	266 8
γ Sagittarii ...	...	...	3.07	- 6 59	269 52
Vega ...	...	...	0.14	+ 61 44	283 56
Altair ...	...	...	0.89	+ 29 19	300 22
β Capricorni ...	...	...	3.25	+ 4 35	302 39
δ Capricorni ...	...	...	2.98	- 2 35	322 8
ε Aquarii ...	...	...	3.83	+ 8 5	324 44
α Aquarii ...	...	...	3.19	+ 10 41	331 57
Fomalhaut ...	...	...	1.29	- 21 7	332 27
λ Aquarii ...	...	...	3.84	- 0 23	340 11
β Piscium ...	...	...	4.58	+ 9 3	347 12
α Pegasi ...	...	...	2.57	+ 19 26	352 4

TABLE II.

Constellation.	Longitude of First point. 1900 A.D.	Length of Constellation on Ecliptic.	Day of Sun's entry into Constellation (present time).	Dates of Coincidence of First Point with	
				Equinoxes. <i>Vernal.</i>	Solstices.
Aries ...	28	22	Apr. 18	110 B.C.	
Taurus ...	50	38	May 11	1680	<i>Summer.</i>
Gemini ...	88	27	June 20	4410	2040 A.D.
Cancer ...	115	20	July 18	6340	110
Leo ...	135	38	Aug. 8	<i>Autumnal.</i>	1330 B.C.
Virgo ...	173	45	Sept. 16	2400 A.D.	4050
Libra ...	218	22	Nov. 1	820 B.C.	7270
Scorpio ...	240	27	Nov. 23	2400	<i>Winter.</i>
Sagittarius ...	267	33	Dec. 19	4330	2120 A.D.
Capricornus ...	300	24	Jan. 20	6700	250 B.C.
Aquarius ...	324	22	Feb. 13	<i>Vernal.</i>	1970
Pisces ...	346	42	Mar. 6	2900 A.D.	3550
Aries ...	28	22	Apr. 18	110 B.C.	6560

It will be seen that the year 2540 B.C. corresponds to a time when the vernal equinox fell far within the limits of *Taurus*, since that date implies a precession of 62° before the present time, whilst the longitude of *Alcyone*, now the *lucida* of the *Pleiades*, is $58^\circ 35'$, and the longitude of *Aldebaran*, the chief

star of *Taurus*, only $68^{\circ} 23'$. The actual date when the equinox passed from *Taurus* into *Aries* was much more nearly 1700 B.C.

But it is one thing to say that the equinoctial point passed from *Taurus* to *Aries* about 1700 B.C. and quite a different thing to say that at this date *Aries* was recognised as having superseded *Taurus* as the constellation that led the year. As a matter of fact, *Aries* did not lead it in 1700 B.C., for, whilst the equinoctial point moves amongst the stars in the direction of diminishing longitude, the Sun in its apparent course through the year moves in the reverse direction. Consequently in 1700 B.C. the Sun was moving, not through *Aries*, but through *Taurus* during the whole of the month succeeding the equinox. This was the epoch when the equinox was at the first point of *Taurus*, just precisely as in the days of Hipparchus it was at the first point of *Aries*. Further, since the bright and distinctive stars of the *Ram* are all at the western end of the constellation, it could not be until the colure had receded as far as Hamal, the *lucida* of the constellation, that there would have been any obvious or pressing necessity for the ancient astronomers to look upon *Aries* as the constellation that opened the year. Now the colure passed through Hamal about 700 B.C. This date then—700 B.C., not 2540 B.C.—is the earliest date when the first month of the year can have been regarded as corresponding with the zodiacal constellation *Aries*, if we accept the assumptions, ordinarily made, that the year began with the vernal equinox, and that the successive constellations of the zodiac were connected with the successive months of the year by the position of the Sun amongst the stars.

The question therefore arises, whether these assumptions are justified, and we have to inquire: (1) Were the successive months connected with the successive constellations either in Accadian or early Semitic calendars? (2) How did the ancient astronomers ascertain the position of the Sun with respect to the stars? (3) Did the ancient astronomers connect the beginning of the year with the equinox, and if so, in what did the connexion consist? With regard to the first inquiry it is curious to note that some popular writers on antiquities systematically write as if it were an established fact both that the constellation figures owed their origin to the climatic conditions of the successive months of the calendar, and that those months owed their names to the constellation figures—two mutually exclusive propositions. Clearly there are only three possible hypotheses: (a) The constellations originated first, and the months and their names were derived from them. (b) The months came first, and from their characteristics the zodiacal constellations were designed afterwards. (c) The two were independent in origin. Any connexion therefore that has been established between them must have arisen in later times.

Of the first of these hypotheses we can dispose at once. It is not a thinkable proposition to assume that any nation devoted

itself to such a systematic study of the starry heavens as to effect the designing of the constellations when as yet it had not progressed sufficiently far to establish for itself a working calendar. Further, the constellations themselves show clearly that they are not more than 5000 years old. It is impossible to conceive that the months of the year and their characteristics had not been recognised prior to that date.

The second hypothesis is not, like the first, impossible upon its very face, but it has serious difficulties. The first is the great irregularity in the length of the zodiacal constellations on the ecliptic. If they were mapped out with a deliberate design to make them correspond to the several months of the year, it seems quite impossible that such a constellation as that of *Virgo* should have been planned to extend over 45° of longitude; still more that it should have been made to follow immediately upon another constellation almost as extended—viz. *Leo*—with 38° of longitude. This objection would have weight if the ancient year had been a solar tropical year, composed of precisely twelve months approximately equal in length. But as the primitive months were lunar months, and the year sometimes contained twelve and sometimes thirteen, the objection gains additional force. That the months of the Assyrian calendar were lunar months, and the year a luni-solar year, is apparent from a great number of tablets. About this there is no doubt. Thus the late Mr. George Smith refers us to *C.I.*, vol. iii. p. 51, lines 7-13, for the translation of a tablet:

“On the 27th day the Moon was visible; on the 28th day, the 29th day, and 30th day observations for an eclipse were made; but it passed, and an eclipse did not take place; the first day when the Moon was seen, the day of the month Tammuz fixed.”

The observations of eclipses, those of the Moon falling on the 14th or 15th day of the month, those of the Sun on the 29th or 30th, show that the months were lunations, commencing with the first actual observation of the new Moon. Thus Mr. George Smith comments on the above tablet:—

“The Assyrian year consisted of twelve lunar months, each month commencing on the evening when the new Moon was first observed; or, in case the Moon was not observed, the new Moon started 30 days after the last month. The months were all thus either of 29 or 30 days, and these two numbers generally but not always alternated.”

Similarly Mr. R. C. Thompson (*Astrological Reports*, p. xix) writes:

“We know from several sources that the Assyrian year consisted of twelve months, each containing usually thirty days, and an intercalary month called the ‘Second Adar’ (the Ve’Adar

of the Hebrews). The Babylonian year consisted of the same number of months, but two intercalary months, Elul and Adar, were added. The examination of the texts, edited in this volume, indicates that the Assyrians, like the Babylonians, had a year composed of lunar months, and it seems that the object of the astrological reports, which relate to the appearance of the Moon and Sun, was to help to determine and foretell the length of the lunar month. If this be so the year in common use throughout Assyria must have been lunar. The calendar assigns to each month thirty full days; the lunar month is, however, little more than twenty-nine and a half days; therefore some of the calendar months must consist of twenty-nine days only. In the report of Balasi we read: 'When the Moon does not wait for the Sun' such and such things will happen. The prediction is followed by the words, which are clearly those of the astrologer, 'It appeared on the fifteenth day with the Sun.' 'When the Moon is not seen with the Sun on the fourteenth day of Adar' such and such things will happen. The prediction is followed by the words, which are again clearly those of the astrologer, 'The day will complete Nisan.' Since the Moon appeared without the Sun on the fourteenth, and with the Sun on the fifteenth, the Moon and the Sun will not be in conjunction before the afternoon of the twenty-ninth day; in this case the Moon would not be visible until the first day of the next month. It must be noted that when the astrologer uses the words 'this night' he refers to the eve of the day—i.e. 'last night.' The words 'The day will complete Nisan' refer to the thirtieth day of the month, and we know from other texts that they indicate that the month will contain thirty full days.*

Since the Assyrian year was thus luni-solar, and the months actual lunations, it is very clear that if a particular month corresponded with the passage of the Sun through a given constellation in one year, it would not do so a couple of years later, inasmuch as the year of twelve lunar months is eleven days short of a solar year. Bearing in mind the great irregularity of size of the zodiacal constellations, and that, as shown by Table II., whilst the Sun passes through *Cancer* in three weeks, it takes six and a half weeks to pass through *Virgo* and six to pass through *Pisces*, there can never have been any correspondence except of the very roughest nature between the constellations and the actual lunar months in any year, and even this rough correspondence would be seriously disturbed in the following year, and quite thrown out in the third year. The point will be made quite clear by the following table for the years 1901-3, showing the position of the Sun in the various

* It will be seen that both these observations are observations at sunrise. When "the Moon did not wait for the Sun" it means that she set before the Sun arose. When Sun and Moon were seen together, it means that the Sun had risen before the Moon had set.

zodiacal constellations on the evenings when the new moons were first observable.

TABLE III.

Probable Date of Observed New Moon.	Longitude of Sun.	Constellation.	Longitude in Constellation.	Month.	Length of Months. Days.
1901 Mar. 21	° 1	Pisces	15	Nisan	30
Apr. 20	29	Aries	1	Iyar	29
May 19	58	Taurus	8	Sivan	29
June 17	86	Taurus	36	Tammuz	30
July 17	114	Gemini	26	Ab	29
Aug. 15	142	Leo	7	Elul	30
Sept. 14	171	Leo	36	Tasrit	29
Oct. 13	200	Virgo	27	Marchesvan	30
Nov. 12	230	Libra	12	Cisleu	30
Dec. 12	260	Scorpio	20	Tebet	30
1902 Jan. 11	291	Sagittarius	24	Sebat	29
Feb. 9	320	Capricornus	20	Adar	30
Mar. 11	352	Pisces	6	Nisan	29
Apr. 9	19	Pisces	33	Iyar	30
May 9	48	Aries	20	Sivan	29
June 7	76	Taurus	26	Tammuz	29
July 6	104	Gemini	16	Ab	30
Aug. 5	132	Cancer	17	Elul	29
Sept. 3	160	Leo	25	Tasrit	29
Oct. 2	189	Virgo	16	Marchesvan	30
Nov. 1	218	Libra	0	Cisleu	30
Dec. 1	249	Scorpio	9	Tebet	30
Dec. 31	279	Sagittarius	12	Sebat	29
1903 Jan. 29	309	Capricornus	9	Adar	30
Feb. 28	339	Aquarius	15	Ve-Adar	30
Mar. 30	9	Pisces	23	Nisan	29
Apr. 28	37	Aries	9	Iyar	30
May 28	66	Taurus	16	Sivan	29
June 26	94	Gemini	6	Tammuz	29
July 25	122	Cancer	7	Ab	30
Aug. 24	150	Leo	15	Elul	29
Sept. 22	179	Virgo	6	Tasrit	29
Oct. 21	207	Virgo	34	Marchesvan	30
Nov. 20	237	Libra	19	Cisleu	30
Dec. 20	268	Sagittarius	1	Tebet	29
1904 Jan. 18	297	Sagittarius	30	Sebat	30
Feb. 17	328	Aquarius	4	Adar	30

The chief ground upon which the connexion between the months and the constellation figures has been alleged, has been the meaning of the names of the months in the Accadian and Assyrian calendars. The following table gives the usual interpretations :—

TABLE IV.

No. of Month.	Assyrian.	Accadian.
1	Month of beginning	Altar of righteousness
2	Month of light	The protecting bull
3	Month of bricks	Month of bricks or of twins
4	Month of Tammuz	Fulness of seed
5	—	Fire makes fire
6	Month of the spirit	Errand of Istar
7	Month of commencement	Month of the holy mound
8	The eighth month	The month opposite the foundation
9	Month of the giant	Month of clouds
10	Month of rain	The crossing-of the sea
11	Month of destruction	The curse of rain
12	Month of darkness	Sowing of seed

It will be observed that in the Assyrian calendar there is not one single instance in which the name of the month has in it anything which could have served as a basis for the zodiacal design. The Accadian months give two or three which have been believed to have a zodiacal significance. These are : the second month, "the protecting or propitious bull"; the third month, "the month of twins"; and the sixth, "the month of the errand or message of Istar." If these be correct we should be forced to conclude that these months no longer retained their original names, but had been renamed with special reference to the three constellations, *Taurus*, *Gemini*, and *Virgo*, since these three meanings carry no meteorological or climatic significance, and have plainly been derived for the months from the constellations, and not for the constellations from the months. In particular the meaning "errand of Istar," if the constellation of *Virgo* is really intended, would be proof that the constellation had been formed, and was known, before the month received this name.

As a matter of fact there is considerable room for difference of opinion as to the meaning of the Accadian words in each of these three cases. Colonel C. R. Conder, R.E., has kindly given us what he considers to be the most probable interpretation for the names of the Accadian months. He would render the name of the second month as "herd becoming full." The third month is the month "of bricks," corresponding to the third Assyrian month. For the sixth month Colonel Conder writes : "I think the rendering 'message of Istar' is far-fetched," and he suggests

the meaning "parched earth" or "heated vegetation." His scheme for the substantial meaning of the Accadian month names would be as follows :—

- | | |
|------------------------|-------------------------|
| 1. The lambing month. | 7. The thunder month. |
| 2. The calving month. | 8. Irrigation month. |
| 3. The brick month. | 9. Very cloudy weather. |
| 4. The harvest month. | 10. Flood month. |
| 5. The very hot month. | 11. Very rainy month. |
| 6. The dried-up month. | 12. Ploughing month. |

It will be seen that this scheme has no zodiacal reference, but it is entirely climatic.

There is therefore no sufficient evidence that the zodiacal constellations owed their origin to the characteristics of the successive months, or, indeed, had any original connexion with them, whatever connexion may have been set up later.

2. How did the ancient astronomers ascertain the position of the Sun with respect to the stars? There are four obvious methods, any one of which might have been employed. The one which we should ourselves make use of nowadays, and therefore the one which is usually supposed to have been employed, is that of the conjunction with certain stars. This relation assumes that men had previously acquainted themselves very thoroughly with the positions of the chief stars near the ecliptic, and had defined the boundaries of the constellations with some precision. By noting what stars rose heliacally on the morning of a given day, and what set heliacally on the evening of the same day, it would be easy for the ancients, supposing they had already provided themselves with fairly correct maps of the stars, to have inferred what were the stars with which the Sun was then in conjunction.

But the employment of this method involves the previous making of the star maps and the actual observation both of certain heliacal risings and heliacal settings. It is simpler to suppose that one or other of these two relations was employed by itself as connecting the Sun with the stars. They required no instruments nor refinements of methods, no star maps, and so may be considered as likely to have been primitive. But the mere existence of the zodiac is sufficient to show that neither the method of heliacal rising nor that of heliacal setting was the one that was actually used. The relation given by either would be a relation to certain particular stars, and the brightness of the stars chosen would be of far more consequence than the fact that they lay within the zodiacal zone. Stars like *Sirius*, *Procyon*, *Betelgeux*, *Capella*, *Arcturus*, would inevitably be associated with the months rather than the zodiacal figures. Such a constellation as *Cancer*, which at best only claims twenty days out of the year, instead of its due proportion of thirty,

would probably have been eliminated altogether, its area being so small, and its stars so inconspicuous.

The fourth method which may have been adopted is the method of opposition—only this would have demanded some mechanical means for determining the passage of time in order that the moment of midnight might be fixed—and the construction and use of such instruments must have been late rather than early.

There is, however, a simple method by which the passage of the Sun through the several constellations might have been watched; a method which demands no instruments, and which does not assume that the early astronomers had furnished themselves with star maps. If month by month, when the new Moon was first seen in the evening sky, the constellation in which the Moon was then seen to be was considered as the one in which the Sun was, the method would be as simple and effective as could be desired.

This method would be substantially that of heliacal setting, but with a difference. If the first new Moon of the year was observed to set at the same time as a bright star, say *Aldebaran*, then twelve lunations later, when the new Moon was again observed to be setting, *Aldebaran* would still be several degrees high, and in the ordinary way Moon and star would set together, not on the first, but on the second evening of the month, since the sidereal year is longer by eleven days than twelve lunations, and the Moon in twenty-four hours passes over a slightly greater arc than the Sun does in eleven days. When twelve more lunations had passed the Moon and *Aldebaran* would usually set together on the third evening of the month. Twelve lunations later again the two bodies would set together on the fourth evening of the month, which would imply that they would also set together on the first evening of the thirteenth lunation. In other words, speaking generally, when Moon and star set together on the third evening of a given month they would set together on the first evening thirteen lunations later, *i.e.* that year would have thirteen months. This method of fixing the calendar is not one that has been handed down to us by tradition, nor does it seem to have been used in later or Western astronomy, but it does appear to have been an Accadian method. For Professor Sayce and Mr. Bosanquet give in the *Monthly Notices* of this Society, vol. xxxix., the following translation of an Accadian inscription:—

“When, on the first day of the month Nisan, the star of stars (or *Dilgan*) and the Moon are parallel, that year is normal. When, on the third day of the month Nisan, the star of stars and the Moon are parallel, that year is full.”

Professor Sayce and Mr. Bosanquet give no reference for the tablet, and we have not been able to trace it. But in all probability their translation is practically correct, for the method

indicated is just one of those eminently simple and practical ones which experience will dictate, but which is not likely to be thought out theoretically. It is clear that the tablet itself afforded to Professor Sayce and Mr. Bosanquet the first suggestion they had ever had of the method, the suitability of which they only ascertained after the tablet had pointed it out to them. They identified *Dilgan*, "the star of stars," with *Capella* by means of a tablet in the Semitic language, which reads :—

"The appearance at the beginning of the year of the star
Icu . . . one observe."

And again :

"The star *Icu* in the month Nisan was seen."

These they take, and no doubt correctly, as being observations of the heliacal rising of *Capella*, and they point out that it rose heliacally at the time of the spring equinox about 2000 B.C., and, further, that its heliacal rising took place before its heliacal setting. These observations they join with the foregoing as together furnishing the determination of the beginning of the year. But the heliacal rising of a star necessarily gives a purely solar year, whilst the setting together of new moon and star gives a luni-solar year, so that the two methods are really incompatible. In both cases the year would be sidereal and not tropical ; and yet not precisely a sidereal year, since precession would slowly alter the declination of the star and change its position for rising and setting.

The stars which would be suitable for such an observation are the following : *Castor*, *Pollux*, and *Procyon* about 4000 B.C. ; γ *Geminorum* and *Sirius* about 2500 B.C. ; *Capella* and β *Tauri* about 2000 B.C. ; *Betelgeux* about 1600 B.C. ; *Aldebaran* about 700 B.C. ; and the *Pleiades* about 200 B.C. In all probability *Capella* was the star in use : it is much the brightest of the stars named, except *Sirius*, which is about 40° distant from the ecliptic. The result of determining the beginning of the year by this method, but using stars other than *Capella*, may be seen from Table V., which gives the mean date relative to the spring equinox of the first day of the first month for different stars, and at different epochs.

TABLE V.

Approximate number of days before or after the spring equinox for the mean beginning of years determined by the seleniacal setting of stars.

Star.	B.C. o. Days.	B.C. 1000. Days.	B.C. 2000. Days.	B.C. 3000. Days.	B.C. 4000. Days.
α <i>Arietis</i>	— 19	...	...	...	...
η <i>Tauri</i>	+ 2	— 12	— 26	...	...
β <i>Orionis</i>	+ 7	— 6	— 19	...	...
<i>Aldebaran</i>	+ 9	— 4	— 18	...	...

Star.	B.C. o. Days.	B.C. 1000. Days.	B.C. 2000. Days.	B.C. 3000. Days.	B.C. 4000. Days.
Betelgeux	+ 23	+ 11	- 3	- 16	- 30
β Tauri	+ 27	+ 12	- 3	- 17	- 31
Capella	...	+ 18	+ 1	- 14	- 28
Sirius	+ 25	+ 14	+ 3	- 8	- 20
Procyon	...	...	+ 21	+ 9	- 3
Castor	...	...	+ 30	+ 14	0
Pollux	...	...	+ 30	+ 14	0

Whenever the method originated, if *Capella* was *Dilgan*, the Sun must have been passing through the constellation *Taurus* during the whole of the first month of the year, so that if any connexion between the signs of the zodiac and the months of the year was then recognised, the *Bull* must have been regarded as the first sign, and the *Ram* as the last; and the connexion between *Taurus* and the first month of the year would have been the more natural, since the second brightest star of the constellation, β *Tauri*—*Alnath*, "*The Horn-Push*"—would have set together with the new moon and *Capella* at the opening of the first month of the year. And so long as this method remained in force the *Bull* must have continued to lead the zodiac, for the beginning of the year necessarily remained always attached to the same star, not to the equinoctial point. The effect of precession therefore would not be to disturb the relation between the beginning of the year and the constellation, but between the beginning of the year and the equinoctial point, so that it would slowly fall later and later, moving from the spring equinox towards the summer solstice. But though we can now see clearly that this forms a serious objection to the method, the early astronomers cannot possibly have known that there was a difference between the two kinds of years until the effect of precession, becoming in the course of centuries too great to be overlooked, forced itself upon their notice. Sooner or later the farmers, even if the astronomers did not, must have become aware that "the times were out of joint," and this method must have fallen into discredit and disuse.

If *Capella* was the "*Dilgan*"* of this method a curious result follows. One month later the new Moon at setting would find itself in almost the same position with regard to two bright stars, namely, *Castor* and *Pollux*, which it had held at the beginning of the first month with *Capella*. If it set with *Capella* the first day of the first month, it would as a rule set with *Castor* and *Pollux* on the thirtieth day afterwards, and so on. Thus *Gemini* would be marked out as connected with the

* As bearing upon this method Colonel Conder's rendering of DIL=iddu, or "times," and GAN=khasisu, or "determine," so that DIL-GAN="determining times," is very appropriate.

second month, just as *Taurus* was associated with the first. We are not aware that any tablet has yet been found in which it is recorded that the beginning of the second month of the year was observed in connexion with the seleniacal setting of *Castor* and *Pollux*, as the first month was with the seleniacal setting of *Capella*. But there are two circumstances which strongly suggest that this relationship had been fully noted. First the different months had their tutelary deities; the second month had two, *Istar* and *Tammuz*, the Heavenly Twins. Next on the *Kudurru* stones, set up as landmarks in Babylonian fields, many curious figures are found, some obviously zodiacal, as, for instance, *Sagittarius*, *Capricornus*, and *Scorpio*. These vary in number and order on different stones, but the two first are almost always the same. The first is a crescent on its back, the second a pair of stars; and these are assigned to the patron deities of the first two months—the Moon-god and the Heavenly Twins. The appropriateness of the first sign to the first month of the year as determined by the method we have described is evident, since at the spring equinox the new Moon when setting is more completely “on its back” than at any other time of the year; and the appropriateness of the second symbol to the twin stars *Castor* and *Pollux*, setting together at the beginning of the second month with the new Moon, is not less clear. The same three symbols are shown on a very interesting tablet now in the British Museum, which is sculptured with a scene representing the worship of the Sun-god in the temple of Sippar, and inscribed with a record of the restoration of the temple by Nabupal-idinna, King of Babylonia about 870 B.C. It is thought that the sculpture, though thus made in the ninth century B.C., was probably reproduced from a much earlier one, as the temple had undergone restoration about two centuries earlier still. The inscription over the symbols is rendered by Colonel Conder:—

“The Moon-god, the Sun-god, and Istar, dwellers in the abyss,
Announce to the years what they are to expect”;

an oracular statement, no doubt having an astrological significance, but none the less very appropriate to the symbols of the first two months, which by their relative positions indicated in advance whether the year was to have an intercalary month or not.

3. The above method had no necessary connection with the equinox. It may have originated at a considerably earlier date than the year 2000 B.C., given above, but in this case the mean date for the beginning of the year must have fallen considerably before the actual equinox. It may have continued long after in use, perhaps even as much as a thousand years. In this case the mean date for the beginning of the first month of the year will have been a fortnight after the equinox. But sooner or later the divergence of the first month from its true position must have

been too great to be tolerated, and there certainly came a time when in one way or another the equinox was observed directly, and the lunar year was corrected to fit the tropical year, and not to fit the sidereal. Thus we read in the tablet numbered in the British Museum as K 15 :

“On the sixth day of the month Nisan the day and the night were equal. The day was six *kasbu*, and the night was six *kasbu*. May Nabu and Marduk be propitious unto the King my lord.”

A similar report, translated by Professor Sayce in vol. iii. of the *Transactions of the Society of Biblical Archaeology*, gives for the equinox the 15th day of Nisan. These two tablets, from the mound of Kouyunjik, are probably of the seventh century B.C., but cannot be earlier than the eighth. They are sufficient to show that at this date the beginning of the year was directly connected with the equinox, but they leave several questions unanswered. The form of the expressions used suggests that some form of time-measurer, probably a sand-glass or water-clock, was employed. We do not know what phase of sunrise and sunset were taken as defining the beginning and end of the day respectively. If the interval from the appearance of the upper limb of the Sun at sunrise to the disappearance of the upper limb at sunset was taken as defining the day, whilst from that disappearance till the following sunrise was taken as defining the night, it would throw the apparent spring equinox two days early in the year, and the apparent autumnal equinox two days late, reducing the winter half of the year to 177 days and increasing the summer half to 188. On the other hand, the irregularities on the actual horizon would tend to shorten the day and to lengthen the night, and so to bring the observed equinox nearer to the true.

A much more important point which is left undetermined is the question as to what was the connexion between the month Nisan and the equinox ; whether Nisan was so taken that the equinox always fell within it, or whether the mean date of the 1st of Nisan was made roughly coincident with the equinox. The difference is important, since the former method would imply that the mean date for Nisan 1 fell about a fortnight before the equinox, and this again would mean that the month Nisan might have been taken to correspond with *Aries* a thousand years earlier than it would have done if the mean date of Nisan 1 had corresponded with the true equinox, or 1200 years earlier, if we suppose that the Assyrians made the three days' error in their determination of the equinox suggested above.

Mr. George Smith states explicitly that “the Assyrian year commenced at the vernal equinox, the new Moon next before the equinox marking the commencement of the new year, the equinox thus falling some time during the first month, Nisan.” But

apparently he bases this conclusion simply on the two equinox tablets cited above, neither of which are conclusive on the matter, as they would equally be consistent with Nisan 1 being taken as the *nearest* new Moon to the equinox, as in the Hebrew calendar. Further on Mr. Smith expresses his opinion that the intercalary months, rendered necessary by the defect of eleven days which a year of twelve lunations shows in each tropical year, were applied in every eighth year, when three months were intercalated. It need scarcely be pointed out that this arrangement, in itself very improbable, would render it impossible for the equinox to fall in Nisan in four years out of every seven.

We do not know of any early evidence bearing on the question of the exact connexion of Nisan with the equinox. There may be other tablets which would enable this point to be settled inferentially. For example, the dates of the Eponym Canon have been fixed by the eclipse of the Sun, which took place in the eponymy of Sagali in the month Sivan. This has been taken as the total eclipse of the Sun which took place on 763 B.C. June 15, and which was a very large partial eclipse at Nineveh. If this identification is accepted, the equinox of that year must have fallen about the ninth day of the month Nisan. This date, like the two others, is therefore inconclusive. It should, however, be pointed out that though the eclipse of 763 B.C. June 15 was very nearly total at Nineveh, and therefore may well have been the eclipse of Sagali, it is not absolutely certain that it was so.* The annular eclipse of 809 B.C. June 13 and the total eclipses of 744 B.C. June 15 and of 716 B.C. June 6 were all of them visible at Nineveh as considerable partial eclipses. It must be remembered that the Assyrians recorded this eclipse, not because they had computed that it was slightly the largest visible at Nineveh during any term of years, but simply because they had actually seen it. There is no reason why the eclipse of 809 B.C. should have been more easily overlooked than that of 763 B.C.†

Whichever of these eclipses we adopt it leaves the question indeterminate as to the precise relation of Nisan 1 to the equinox, and so also does an observation cited by Mr. George Smith of the occurrence of the longest night on the 15th day of the 10th month. The equinox in this case must have fallen about the fifth day of Nisan, and in order to settle the question in

* The reasoning by which Dr. Oppert has been led to fix on B.C. 809, and Mr. George Smith on B.C. 763, as the date of the eclipse of Sagali, implies that the seleniacal setting of *Capella* was then no longer used to determine the beginning of the month Nisan. If that method had been still in use, the eclipse of B.C. 820 July 15, or that of B.C. 803 August 6, would have to be substituted for the foregoing; since in that case Sivan 29 might fall in July or the first week of August, but could not fall in June.

† Or the reason of this eclipse being alone recorded of all those visible at Nineveh during the years covered by the Eponym Canon may have been that it coincided with a serious "revolt in the city of Assur." Eclipses that did not synchronise with any great national disaster may well have gone unnoted.

dispute it would be necessary to find a year in which the equinox had fallen either late in Adar or Ve-Adar, or in the last fortnight of Nisan. The point is one of first importance, for, as already indicated, it means a difference of a thousand years in the possible adoption of the Ram Zodiac. In the one case this may have taken place as early as 1700 B.C., or, allowing for the suggested error in the determination of the equinox, 1900 B.C.; whilst in the other case the earliest possible dates would be 700 B.C. or 900 B.C., according as the true or apparent equinox was observed.

But though there is this ambiguity about these earlier records, it seems quite clear that in later times the first day of Nisan could fall after the spring equinox. Fathers Epping and Strassmaier have translated and discussed three Babylonian calendars, and from the eclipses mentioned in them have been able to deduce the Julian date for Nisan 1 in five years.

Year of the Seleucidæan Era.	Year B.C.	Julian date of Nisan 1.
188	124	April 4
189	123	March 25
190	122	April 12
201	111	April 10
202	110	March 30*

It will be seen that in each of these five instances Nisan 1 was later than the true spring equinox, which fell at that time on March 24, so that in the second century before our era the true equinox must have been observed and the day of the nearest new moon, whether before or after it, must have been taken in general as the first day of Nisan.

Even if we did not possess this direct evidence, we should be still strongly inclined to adopt the latest date, for it is quite contrary to experience to suppose that a method of determining the beginning of the year, so simple and effective as that of the seleniacal setting of *Capella*, would be abandoned in favour of a method on an entirely different principle before any inconvenience had arisen from the inherent defects of the first method. Men are far too conservative to make a change of this sweeping character until the pressure of necessity is brought to bear upon them.

Whenever the change took place, and however it was effected, it was nothing less than a complete revolution. The old "star of stars" was abandoned; the primacy of the zodiac was transferred from *Taurus* to the constellation which had previously been regarded as the last of all, *Aries*; the mean date of the beginning of the year was shifted earlier by more than three weeks; and the use of instrumental means of observation of one

* *Astronomisches aus Babylon*, p. 39.

kind or another was introduced in place of the simple scrutiny of the heavens. In the earlier age of astronomy, the age of *Taurus* and *Capella*, we have as yet no direct evidence of observation of the planets. In the second age, the age of *Aries*, the labours of Fathers Epping and Strassmaier have shown us that under the Seleucidæ and Arsacidæ not only were the places of the Moon and five planets carefully observed, but they were also predicted. But there is reason to think that much earlier than this, four or five centuries earlier, the movements of the planets had been carefully watched, and certain relations recognised—at least in the case of the planet *Jupiter*.

If the path of an exterior planet be watched it will be seen to come to a stationary point and then to retrograde for a considerable time, coming to a second stationary point, and afterwards resuming its forward movement. Midway between the two stationary points the planet is in opposition. Consequently, when once the apparent movements of the planets had become familiar to the observers and the period of retrogression for each was known it was only necessary to watch for the time when a planet became stationary as a morning star, and then it would be known that half the number of days of retrogression later it would be in opposition. There are three exterior planets known to the ancients, *Mars*, *Jupiter*, and *Saturn* : of these three *Mars* comes to opposition but once in somewhat over two years, and the time during which it is favourably placed for observation is comparatively short. *Saturn*, on the other hand, moves very slowly and is not conspicuous. *Jupiter*, the remaining planet and the brightest of the three, seems exactly fitted to draw attention. Its average rate of motion is 1° in four days—half the diameter of the Moon in twenty-four hours. This is an amount that the eye can readily detect, and with a little practice the ancient astronomers would no doubt soon have been able to fix the time of the stationary point of *Jupiter* with a fair degree of accuracy. The interval from this date to the time of opposition would be almost precisely two lunar months, and two lunar months again would bring it to its second stationary point. It would then be noted that from one opposition to the next *Jupiter* would pass through almost exactly one-twelfth part of the ecliptic—that is to say, the mean length of a constellation of the zodiac. Further, the arc traversed during the four months of retrogression would be, roughly, the third part of the mean length of a zodiacal constellation. In other words, it covered the arc which in later times was known as a “decan”—an arc, that is to say, of 10° . The march of *Jupiter* in a year corresponded therefore, in the mean, to the march of the Sun in a month ; and hence no doubt it was that Marduk, the supreme god of the Babylonian pantheon, was intimately connected both with the Sun and with the planet *Jupiter*.

In the fifth tablet of the Babylonian Story of the Creation, translated by Mr. L. W. King, there seems a clear reference to

this action of the planet *Jupiter* as the divider of the ecliptic. The tablet runs:

“He” (*i.e.* Marduk) “made the stations for the great gods;
The stars, their images, as the stars of the zodiac he fixed.
He ordained the year and into sections he divided it;
For the twelve months he fixed three stars.
After he had . . . The days of the year . . . images.
He founded the station of Nibur to determine their bounds;
That none might err or go astray.
He set the station of Bel and Ea along with him.”

The division of the year by Marduk into twelve parts and the allotment of three “stars” to each part corresponds to the division of the ecliptic by the planet *Jupiter* into twelve sections, each of which it subdivides again into three “decans.” But such a division implies that already the old irregular partition of the ecliptic between the zodiacal constellations was being superseded by a division into twelve equal parts, each composed of three portions of ten degrees in length. The works of Autolycus the Æolian in the fourth century B.C. are the earliest extant in which this division into the “dodecatomoria” is explicitly mentioned.

The significance of the line “He founded the station of Nibur” lies in the fact that different names were applied to the planet *Jupiter* under different circumstances. Thus a different name was given to the planet according to the month of the year in which it was observed, and it became *Nibiru* in the month Tisri, that is to say, at the autumnal equinox. The name *Nibiru* was also given to it when it culminated, “When it stands in the meridian it becomes Nibiru” (*Astrological Report*, No. 94, in R. C. Thompson’s translation). Thus the founding of the station of Nibur by Marduk is the equivalent of the planet *Jupiter* marking out the meridian and the ascending node. It is in complete accord with this meaning that Colonel C. R. Conder explains *Nibiru* as the “crossing over”; for the meridian is the crossing place for a celestial body from its ascent to its descent, and the ascending node is the crossing place from the southern hemisphere to the northern. The distinct recognition of the meridian, of the ecliptic and of the equator, of the nodes of these two great circles, and of the position of *Jupiter* as in opposition to the Sun, shows a great advance in the science of astronomy upon the times when the seleniacal setting of stars formed the chief fundamental observation. This knowledge does not appear to have reached the Ionian Greeks until about 600 B.C., when the recognition of the obliquity of the ecliptic and the division of the celestial sphere into five zones are ascribed to Thales of Miletus, or to his pupil Anaximander.

We can dimly discern from the foregoing considerations that primitive astronomy had passed through two very distinct and

well-defined epochs before the time of Hipparchus. To the first age, the age in which the *Bull* led the zodiac, we must assign the formation of the constellations, which cannot have taken place earlier than 3000 B.C., nor much later than 2500 B.C. In this age, amongst the Accadians at least, the beginning of the year was determined by the setting together of *Capella* and the new Moon. The second age was that in which the *Ram* was recognised as the leader of the celestial host, and it probably began not earlier than 700 B.C., possibly even later. It was the age in which the equinox was determined by instrumental observation, *i.e.* by the use of a mechanical time-measurer. It was the age also of the recognition of the apparent motions of the planets—at any rate of the planet *Jupiter*—and before it closed the regular observation of the planets had reached a remarkably high state of development. Its rise was probably connected very intimately with the great outburst of literary activity which characterised the reign of Assur-bani-pal. The first age, the age of the Bull Zodiac, cannot

Ephemeris for Physical Observations of

Greenwich Mean Noon.	P.	L-O.	B.	A-L.	B.	Q.	d.
1904.							
Nov. 7	23°45	267°92	+25°26	-30°35	+21°67	294°46	28°06
9	24°09	269°23	25°24	30°75	21°87	294°48	28°37
11	24°72	270°54	25°21	31°14	22°06	294°49	28°68
13	25°33	271°85	25°17	31°53	22°25	294°49	28°99
15	25°94	273°15	25°12	31°91	22°43	294°48	29°29
17	26°53	274°45	25°05	32°29	22°61	294°47	29°60
19	27°11	275°74	24°98	32°65	22°78	294°45	29°89
21	27°68	277°03	24°90	33°00	22°95	294°42	30°17
23	28°23	278°31	24°80	33°34	23°11	294°38	30°45
25	28°77	279°58	24°68	33°68	23°26	294°32	30°74
27	29°30	280°84	24°56	34°01	23°41	294°26	31°02
29	29°81	282°10	24°43	34°33	23°55	294°19	31°29
Dec. 1	30°31	283°35	24°28	34°64	23°69	294°12	31°56
3	30°80	284°59	24°12	34°94	23°82	294°04	31°83
5	31°27	285°83	23°96	35°24	23°95	293°95	32°11
7	31°72	287°06	23°79	35°52	24°07	293°85	32°37
9	32°16	288°28	23°61	35°80	24°18	293°74	32°64
11	32°59	289°49	23°42	36°06	24°29	293°63	32°89
13	33°00	290°69	+23°22	-36°30	+24°39	293°51	33°12

have passed gradually by insensible degrees into the second age, the age of the Ram Zodiac. The threefold change,—of the initial constellation, of the method of determining the beginning of the year, and of the relation of the year to the actual seasons by an interval of three weeks or more,—must have entailed a complete astronomical revolution.

The third age we may consider as beginning with Hipparchus. It brought the recognition of precession, and in consequence the actual "constellations" of the zodiac were superseded by the "signs" of the zodiac; that is to say, for the actual groups of stars were substituted purely arbitrary, imaginary, and equal divisions of the ecliptic. In this age the apparent motions of the planets were not only noted, but under the Ptolemaic hypothesis were reduced to a definite mathematical system. For the fourth age men had to wait until the days of Galileo brought the acceptance of the Copernican doctrine and the invention of the telescope.

Mars, 1904-5-6. By A. C. D. Crommelin.

Greenwich Mean, Noon.	☉	Light- time.	Appar. Diam.	Defect of Illumi- nation.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24 ^h +.
1904.	m	"	"	"	"	h m	m
Nov. 7	60° 10	17.249	4.48	0.26	312.23	3 16.40	40.10
9		17.121	4.51	.27	292.72	4 36.59	
11		16.991	4.55	.28	273.23	5 56.71	
13		16.859	4.58	.29	253.74	7 16.83	
15		16.725	4.62	.30	234.26	8 36.91	
17		16.589	4.66	.30	214.78	9 56.99	
19	65.34	16.452	4.70	.31	195.31	11 17.02	40.02
21		16.312	4.74	.32	175.84	12 37.05	
23		16.171	4.78	.33	156.38	13 57.04	
25		16.028	4.82	.34	136.93	15 16.99	
27		15.884	4.87	.35	117.48	16 36.94	
29		15.739	4.91	.36	98.04	17 56.85	
Dec. 1	70.58	15.592	4.96	.37	78.61	19 16.72	39.92
3		15.444	5.01	.38	59.19	20 36.54	
5		15.294	5.06	.39	39.77	21 56.37	
7		15.143	5.11	.40	20.36	23 16.15	
9		14.990	5.16	.41	0.96	24 35.89	
11		14.836	5.21	.42	341.58	1 15.71*	
13	75.82	14.681	5.27	0.43	322.20	2 35.37	39.82

Greenwich
Mean
Noon.

	P.	L-O.	B.	A-L.	B.	Q.	d.
^{1904.} Dec. 15	33°40	291°88	+23°01	-36°54	+24°49	293°38	33°37
17	33°78	293°06	22°79	36°76	24°58	293°23	33°60
19	34°14	294°23	22°57	36°97	24°67	293°08	33°83
21	34°49	295°39	22°34	37°17	24°75	292°93	34°06
23	34°82	296°54	22°10	37°36	24°82	292°77	34°28
25	35°14	297°68	21°85	37°53	24°89	292°60	34°49
27	35°44	298°81	21°60	37°70	24°95	292°43	34°70
29	35°73	299°93	21°34	37°85	25°00	292°26	34°90
31	36°01	301°03	21°07	37°99	25°05	292°08	35°09
^{1905.} Jan. 2	36°27	302°12	20°80	38°11	25°09	291°90	35°27
4	36°51	303°20	20°52	38°22	25°12	291°71	35°44
6	36°74	304°27	20°24	38°32	25°15	291°51	35°60
8	36°95	305°33	19°95	38°40	25°17	291°30	35°75
10	37°15	306°37	19°66	38°47	25°19	291°08	35°89
12	37°34	307°40	19°37	38°52	25°20	290°86	36°02
14	37°51	308°41	19°07	38°56	25°21	290°64	36°14
16	37°67	309°41	18°77	38°58	25°20	290°41	36°24
18	37°82	310°40	18°47	38°59	25°19	290°17	36°33
20	37°95	311°37	18°17	38°58	25°18	289°92	36°41
22	38°07	312°33	17°87	38°56	25°16	289°67	36°48
24	38°17	313°28	17°56	38°53	25°13	289°42	36°54
26	38°26	314°21	17°25	38°48	25°10	289°17	36°59
28	38°34	315°12	16°94	38°41	25°06	288°92	36°62
30	38°42	316°02	16°63	38°32	25°01	288°67	36°64
Feb. 1	38°48	316°90	16°32	38°22	24°96	288°42	36°64
3	38°53	317°77	16°02	38°09	24°90	288°17	36°62
5	38°56	318°62	15°72	37°95	24°83	287°92	36°60
7	38°59	319°45	15°42	37°79	24°75	287°67	36°57
9	38°61	320°26	15°12	37°62	24°67	287°42	36°52
11	38°62	321°05	14°83	37°43	24°58	287°17	36°44
13	38°62	321°82	14°54	37°21	24°49	286°92	36°34
15	38°61	322°57	14°25	36°98	24°39	286°66	36°22
17	38°60	323°31	13°97	36°73	24°28	286°40	36°09
19	38°58	324°02	13°70	36°46	24°17	286°15	35°93
21	38°55	324°71	13°43	36°17	24°05	285°89	35°74
23	38°52	325°38	13°17	35°85	23°93	285°63	35°53
25	38°49	326°02	12°92	35°49	23°80	285°38	35°29
27	38°45	326°64	+12°68	-35°10	+23°66	285°14	35°02

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Greenwich Mean. Noon.	☉	Light- time.	Appar. Diam.	Defect of Illumi- nation.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24 ^h + m
1904.	°	m	"	"	°	h m	
Dec. 15		14 ^h 52 ^m 5	5 ["] 32	0 ["] 44	302 [°] 83	3 54 ^m 97	
17		14 ^h 36 ^m 7	5 ["] 38	45	283 [°] 47	5 14 ^m 53	
19		14 ^h 20 ^m 8	5 ["] 44	46	264 [°] 12	6 34 ^m 06	
21		14 ^h 04 ^m 8	5 ["] 50	47	244 [°] 78	7 53 ^m 54	
23		13 ^h 88 ^m 7	5 ["] 56	48	225 [°] 45	9 12 ^m 98	
25	81 [°] 07	13 ^h 72 ^m 5	5 ["] 63	50	206 [°] 14	10 32 ^m 33	39 ^m 67
27		13 ^h 56 ^m 2	5 ["] 70	51	186 [°] 84	11 51 ^m 64	
29		13 ^h 39 ^m 9	5 ["] 77	52	167 [°] 55	13 10 ^m 91	
31		13 ^h 23 ^m 4	5 ["] 84	53	148 [°] 27	14 30 ^m 14	
1905.							
Jan. 2		13 ^h 06 ^m 9	5 ["] 92	54	129 [°] 00	15 49 ^m 32	
4		12 ^h 90 ^m 3	6 ["] 00	56	109 [°] 74	17 8 ^m 46	
6	86 [°] 34	12 ^h 73 ^m 6	6 ["] 08	57	90 [°] 50	18 27 ^m 52	39 ^m 52
8		12 ^h 56 ^m 8	6 ["] 16	58	71 [°] 27	19 46 ^m 54	
10		12 ^h 40 ^m 0	6 ["] 24	59	52 [°] 06	21 5 ^m 50	
12		12 ^h 23 ^m 1	6 ["] 33	60	32 [°] 86	22 24 ^m 39	
14		12 ^h 06 ^m 2	6 ["] 42	62	13 [°] 67	23 43 ^m 24	
16		11 ^h 89 ^m 3	6 ["] 51	63	354 [°] 49	0 22 ^m 64*	
18	91 [°] 64	11 ^h 72 ^m 3	6 ["] 60	64	335 [°] 33	1 41 ^m 36	39 ^m 35
20		11 ^h 55 ^m 3	6 ["] 70	65	316 [°] 18	3 0 ^m 03	
22		11 ^h 38 ^m 3	6 ["] 80	67	297 [°] 04	4 18 ^m 67	
24		11 ^h 21 ^m 3	6 ["] 90	68	277 [°] 92	5 37 ^m 21	
26		11 ^h 04 ^m 2	7 ["] 00	69	258 [°] 81	6 55 ^m 72	
28		10 ^h 87 ^m 1	7 ["] 11	70	239 [°] 72	8 14 ^m 14	
30	96 [°] 97	10 ^h 70 ^m 0	7 ["] 22	71	220 [°] 65	9 32 ^m 48	39 ^m 17
Feb. 1		10 ^h 52 ^m 9	7 ["] 34	73	201 [°] 59	10 50 ^m 77	
3		10 ^h 35 ^m 9	7 ["] 46	74	182 [°] 55	12 8 ^m 99	
5		10 ^h 18 ^m 9	7 ["] 59	75	163 [°] 53	13 27 ^m 13	
7		10 ^h 01 ^m 9	7 ["] 72	76	144 [°] 53	14 45 ^m 17	
9		9 ^h 85 ^m 0	7 ["] 85	77	125 [°] 54	16 3 ^m 17	
11	102 [°] 36	9 ^h 68 ^m 2	7 ["] 99	78	106 [°] 57	17 21 ^m 08	38 ^m 93
13		9 ^h 51 ^m 4	8 ["] 13	79	87 [°] 63	18 38 ^m 87	
15		9 ^h 34 ^m 6	8 ["] 28	80	68 [°] 71	19 56 ^m 57	
17		9 ^h 17 ^m 9	8 ["] 43	81	49 [°] 80	21 14 ^m 23	
19		9 ^h 01 ^m 3	8 ["] 58	82	30 [°] 91	22 31 ^m 80	
21		8 ^h 84 ^m 8	8 ["] 74	82	12 [°] 04	23 49 ^m 28	
23	107 [°] 79	8 ^h 68 ^m 4	8 ["] 91	83	353 [°] 20	0 27 ^m 95*	38 ^m 67
25		8 ^h 52 ^m 1	9 ["] 08	83	334 [°] 38	1 45 ^m 22	
27		8 ^h 36 ^m 0	9 ["] 25	0 ["] 84	315 [°] 58	3 2 ^m 41	

Greenwich Mean Noon.	P.	L-O.	B.	A-L.	B.	Q.	d.
1905. Mar. 1	38°41	327°24	+ 12°44	- 34°69	+ 23°52	284°90	34°73
3	38°37	327°81	12°22	34°27	23°37	284°67	34°41
5	38°32	328°35	12°00	33°83	23°21	284°44	34°06
7	38°27	328°87	11°79	33°38	23°04	284°21	33°68
9	38°22	329°36	11°60	32°90	22°87	284°00	33°27
11	38°17	329°81	11°42	32°39	22°69	283°81	32°83
13	38°13	330°23	11°26	31°84	22°51	283°63	32°36
15	38°09	330°62	11°11	31°25	22°32	283°46	31°85
17	38°05	330°98	10°98	30°62	22°12	283°29	31°30
19	38°01	331°31	10°87	29°96	21°91	283°13	30°71
21	37°98	331°60	10°77	29°26	21°70	282°98	30°07
23	37°95	331°85	10°69	28°53	21°48	282°85	29°39
25	37°92	332°06	10°63	27°77	21°25	282°73	28°66
27	37°90	332°23	10°59	26°96	21°02	282°62	27°89
29	37°88	332°36	10°57	26°11	20°78	282°52	27°07
31	37°88	332°46	10°56	25°21	20°54	282°44	26°21
Apr. 2	37°88	332°51	10°59	24°27	20°29	282°38	25°29
4	37°89	332°50	10°64	23°29	20°04	282°34	24°32
6	37°91	332°45	10°70	22°27	19°78	282°31	23°30
8	37°94	332°36	10°79	21°21	19°51	282°30	22°23
10	37°97	332°23	10°90	20°10	19°24	282°29	21°10
12	38°00	332°06	11°03	18°94	18°96	282°30	19°92
14	38°04	331°83	11°19	17°73	18°68	282°32	18°67
16	38°10	331°55	11°38	16°47	18°39	282°36	17°36
18	38°15	331°23	11°59	15°16	18°09	282°40	16°00
20	38°21	330°87	11°82	13°82	17°79	282°46	14°61
22	38°27	330°47	12°07	12°45	17°48	282°55	13°18
24	38°34	330°03	12°33	11°05	17°17	282°65	11°72
26	38°41	329°54	12°62	9°57	16°85	282°70	10°18
28	38°48	329°02	12°93	8°05	16°53	282°74	8°60
30	38°55	328°45	13°26	6°51	16°20	282°72	6°98
May 2	38°62	327°87	13°60	4°96	15°87	282°50	5°32
4	38°69	327°26	13°94	3°40	15°53	282°15	3°66
6	38°75	326°63	14°29	1°82	15°18	281°62	1°98
8	38°81	325°99	14°65	- 0°21	14°83	...	0°28
10	38°86	325°33	15°01	+ 1°45	14°47	...	1°51
12	38°90	324°67	15°37	3°10	14°10	106°36	3°23
14	38°94	324°01	+ 15°73	+ 4°73	+ 13°73	105°85	5°01

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Greenwich Mean, Noon.	☉	Light- time.	Appar. Diam.	Defect of Illumi- nation.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24 ^h +.
1905.	°	m	"	"		h m	m
Mar. 1		8 ^h 19 ^m 9	9 ["] 43	0 ["] 84	296°80	4 19 ^m 51	
3		8 ^h 04 ^m 0	9 ["] 62	84	278°05	5 36 ^m 48	
5		7 ^h 88 ^m 3	9 ["] 81	84	259°33	6 53 ^m 31	
7	113°29	7 ^h 72 ^m 7	10 ["] 01	84	240°64	8 10 ^m 02	38 ^m 34
9		7 ^h 57 ^m 3	10 ["] 21	84	221°98	9 26 ^m 60	
11		7 ^h 42 ^m 1	10 ["] 42	83	203°34	10 43 ^m 09	
13		7 ^h 27 ^m 0	10 ["] 64	83	184°74	11 59 ^m 42	
15		7 ^h 12 ^m 1	10 ["] 86	82	166°17	13 15 ^m 62	
17		6 ^h 97 ^m 5	11 ["] 09	81	147°64	14 31 ^m 66	
19	118°87	6 ^h 83 ^m 2	11 ["] 32	79	129°14	15 47 ^m 57	37 ^m 95
21		6 ^h 69 ^m 2	11 ["] 56	78	110°66	17 3 ^m 39	
23		6 ^h 55 ^m 4	11 ["] 80	76	92°22	18 19 ^m 05	
25		6 ^h 41 ^m 9	12 ["] 05	74	73°83	19 34 ^m 50	
27		6 ^h 28 ^m 6	12 ["] 31	72	55°48	20 49 ^m 78	
29		6 ^h 15 ^m 6	12 ["] 57	69	37°17	22 4 ^m 89	
31	124°52	6 ^h 03 ^m 0	12 ["] 83	66	18°90	23 19 ^m 82	37 ^m 44
Apr. 2		5 ^h 90 ^m 7	13 ["] 09	63	0°67	24 34 ^m 60	
4		5 ^h 78 ^m 9	13 ["] 36	59	342°48	1 11 ^m 89*	
6		5 ^h 67 ^m 4	13 ["] 63	56	324°33	2 26 ^m 34	
8		5 ^h 56 ^m 2	13 ["] 90	52	306°23	3 40 ^m 58	
10		5 ^h 45 ^m 6	14 ["] 17	47	288°18	4 54 ^m 60	
12	130°27	5 ^h 35 ^m 4	14 ["] 44	43	270°17	6 8 ^m 46	36 ^m 90
14		5 ^h 25 ^m 7	14 ["] 71	38	252°20	7 22 ^m 15	
16		5 ^h 16 ^m 5	14 ["] 97	34	234°27	8 35 ^m 68	
18		5 ^h 07 ^m 7	15 ["] 22	30	216°39	9 49 ^m 00	
20		4 ^h 99 ^m 5	15 ["] 47	25	198°55	11 2 ^m 14	
22		4 ^h 91 ^m 8	15 ["] 72	21	180°76	12 15 ^m 08	
24	136°12	4 ^h 84 ^m 7	15 ["] 96	16	163°01	13 27 ^m 85	36 ^m 36
26		4 ^h 78 ^m 1	16 ["] 18	13	145°29	14 40 ^m 49	
28		4 ^h 72 ^m 1	16 ["] 38	10	127°60	15 53 ^m 00	
30		4 ^h 66 ^m 7	16 ["] 57	06	109°96	17 5 ^m 31	
May 2		4 ^h 62 ^m 0	16 ["] 74	04	92°36	18 17 ^m 44	
4		4 ^h 57 ^m 8	16 ["] 89	02	74°77	19 29 ^m 54	
6	142°07	4 ^h 54 ^m 3	17 ["] 02	01	57°18	20 41 ^m 64	36 ^m 04
8		4 ^h 51 ^m 4	17 ["] 13	...	39°61	21 53 ^m 65	
10		4 ^h 49 ^m 2	17 ["] 22	...	22°05	23 5 ^m 57	
12		4 ^h 47 ^m 6	17 ["] 28	01	4°52	24 17 ^m 46	
14		4 ^h 46 ^m 6	17 ["] 32	00 ["] 3	346°97	0 53 ^m 42*	

Greenwich Mean Noon.	P.	L-O.	B.	A-L.	B.	Q.	d.
1905.							
May 16	38°97	323°36	+ 16°08	+ 6°34	+ 13°36	105°84	6°71
18	39°00	322°72	16°43	7°94	12°98	105°85	8°42
20	39°02	322°10	16°77	9°53	12°60	106°00	10°12
22	39°03	321°51	17°09	11°11	12°21	106°14	11°82
24	39°04	320°94	17°40	12°67	11°82	106°28	13°48
26	39°05	320°40	17°69	14°19	11°43	105°42	15°10
28	39°05	319°90	17°97	15°66	11°03	106°56	16°65
30	39°05	319°43	18°23	17°08	10°63	106°70	18°18
June 1	39°05	319°01	18°47	18°46	10°22	106°84	19°67
3	39°05	318°65	18°68	19°80	9°81	106°97	21°12
5	39°04	318°34	18°87	21°10	9°39	107°10	22°53
7	39°04	318°07	19°04	22°35	8°97	107°21	23°89
9	39°04	317°85	19°19	23°56	8°54	107°31	25°20
11	39°04	317°67	19°32	24°72	8°11	107°39	26°45
13	39°05	317°57	19°43	25°82	7°68	107°46	27°64
15	39°06	317°52	19°51	26°87	7°24	107°52	28°78
17	39°07	317°52	19°57	27°85	6°80	107°57	29°87
19	39°08	317°57	19°61	28°77	6°35	107°60	30°91
21	39°09	317°67	19°63	29°64	5°90	107°62	31°91
23	39°11	317°82	19°64	30°48	5°45	107°62	32°87
25	39°14	318°02	19°62	31°27	5°00	107°62	33°77
27	39°16	318°28	19°58	32°02	4°54	107°61	34°62
29	39°19	318°58	19°52	32°72	4°08	107°59	35°43
July 1	39°21	318°93	19°44	33°38	3°62	107°56	36°20
3	39°24	319°32	19°34	33°99	3°15	107°52	36°93
5	39°26	319°74	19°22	34°56	2°68	107°46	37°62
7	39°28	320°21	19°08	35°09	2°21	107°39	38°27
9	39°31	320°72	18°93	35°59	1°74	107°30	38°87
11	39°33	321°28	18°76	36°06	1°26	107°21	39°43
13	39°35	321°87	18°57	36°49	0°78	107°10	39°96
15	39°36	322°49	18°37	36°88	+ 0°30	106°98	40°46
17	39°36	323°14	18°15	37°24	- 0°18	106°84	40°93
19	39°36	323°82	17°91	37°58	0°66	106°69	41°37
21	39°35	324°53	17°66	37°90	1°14	106°53	41°79
23	39°34	325°27	17°39	38°19	1°63	106°36	42°17
25	39°31	326°03	17°11	38°46	2°11	106°18	42°52
27	39°27	326°82	16°81	38°71	2°60	105°98	42°85
29	39°22	327°54	+ 16°50	+ 38°95	- 3°09	105°77	43°16

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Greenwich Mean. Noon.	☉	Light- time.	Appar. Diam.	Defect of Illumi- nation.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24 ^h + m
1905.	°	m	"	"	°	h m	
May 16		4'46.2	17'33	0'06	329°39	2 5'53	
18	148°15	4'46.4	17'33	10	311°80	3 17'68	36°07
20		4'47.2	17'29	14	294°20	4 29'86	
22		4'48.6	17'24	19	276°59	5 42'09	
24		4'50.5	17'17	24	258°96	6 54'41	
26		4'53.0	17'07	29	241°26	8 7'02	
28		4'56.0	16'96	35	223°52	9 19'79	
30	154°34	4'59.5	16'83	42	205°75	10 32'69	36°47
June 1		4'63.5	16'68	49	187°95	11 45'71	
3		4'67.9	16'53	56	170°10	12 58'94	
5		4'72.8	16'36	62	152°19	14 12'48	
7		4'78.1	16'17	69	134°22	15 26'27	
9		4'83.7	15'99	76	116°20	16 40'26	
11	160°66	4'89.7	15'79	83	98°14	17 54'44	37°10
13		4'96.1	15'59	89	80°03	19 8'82	
15		5'02.8	15'38	0'95	61°86	20 23'47	
17		5'09.7	15'17	1'01	43°63	21 38'37	
19		5'16.9	14'96	1'06	25°34	22 53'52	
21		5'24.4	14'75	1'11	7°00	24 8'88	
23	167°11	5'32.0	14'53	1'16	348°62	0 46'71*	37°77
25		5'39.9	14'32	1'21	330°19	2 2'38	
27		5'48.0	14'11	1'25	311°70	3 18'30	
29		5'56.2	13'90	1'29	293°16	4 34'44	
July 1		5'64.7	13'69	1'32	274°57	5 50'79	
3		5'73.3	13'49	1'35	255°94	7 7'31	
5	173°70	5'82.0	13'29	1'38	237°28	8 23'96	38°36
7		5'90.8	13'09	1'41	218°57	9 40'81	
9		5'99.7	12'90	1'43	199°81	10 57'90	
11		6'08.8	12'70	1'45	181°02	12 15'12	
13		6'17.9	12'52	1'46	162°19	13 32'51	
15		6'27.1	12'33	1'48	143°34	14 49'98	
17	180°42	6'36.3	12'16	1'49	124°45	16 7'62	38°83
19		6'45.7	11'98	1'50	105°53	17 25'38	
21		6'55.0	11'81	1'50	86°58	18 43'27	
23		6'64.3	11'64	1'51	67°61	20 1'24	
25		6'73.8	11'48	1'51	48°61	21 19'34	
27		6'83.3	11'32	1'51	29°58	22 37'57	
29	187°28	6'92.8	11'16	1'51	10°52	23 55'92	39°18

Greenwich Mean Noon.	P.	L-O.	B.	A-L.	B.	Q.	d.
1905.							
July 31	39°16	328°48	+16°17	+39°16	- 3°58	105°55	43°45
Aug. 2	39°09	329°34	15°83	39°35	4°07	105°32	43°72
4	39°01	330°22	15°47	39°52	4°56	105°07	43°97
6	38°91	331°12	15°10	39°68	5°05	104°81	44°19
8	38°80	332°04	14°72	39°84	5°54	104°54	44°40
10	38°67	332°98	14°32	39°99	6°03	104°25	44°59
12	38°52	333°94	13°91	40°12	6°52	103°94	44°76
14	38°36	334°91	13°48	40°23	7°01	103°62	44°90
16	38°18	335°89	13°05	40°33	7°49	103°29	45°02
18	37°98	336°89	12°61	40°43	7°98	102°95	45°12
20	37°77	337°90	12°15	40°52	8°46	102°59	45°23
22	37°54	338°92	11°68	40°60	8°94	102°22	45°32
24	37°29	339°96	11°20	40°67	9°42	101°84	45°39
26	37°01	341°01	10°70	40°74	9°90	101°45	45°45
28	36°72	342°07	10°20	40°80	10°38	101°04	45°50
30	36°42	343°14	9°69	40°86	10°85	100°62	45°54
Sept. 1	36°09	344°22	9°17	40°91	11°32	100°19	45°56
3	35°74	345°31	8°64	40°97	11°78	99°75	45°57
5	35°37	346°41	8°10	41°02	12°24	99°30	45°57
7	34°97	347°52	7°55	41°07	12°70	98°83	45°56
9	34°56	348°65	6°99	41°12	13°15	98°35	45°54
11	34°13	349°78	6°42	41°16	13°60	97°86	45°50
13	33°67	350°92	5°85	41°20	14°05	97°36	45°46
15	33°20	352°06	5°27	41°24	14°49	96°85	45°41
17	32°70	353°21	4°68	41°28	14°93	96°32	45°35
19	32°19	354°37	4°08	41°32	15°36	95°79	45°28
21	31°65	355°55	3°48	41°36	15°79	95°25	45°21
23	31°09	356°73	2°87	41°39	16°21	94°70	45°12
25	30°51	357°92	2°26	41°42	16°63	94°14	45°03
27	29°92	359°11	1°65	41°46	17°03	93°58	44°93
29	29°30	0°31	1°03	41°49	17°43	93°01	44°83
Oct. 1	28°67	1°52	+0°41	41°53	17°82	92°44	44°72
3	28°02	2°74	-0°22	41°56	18°20	91°86	44°60
5	27°35	3°97	0°85	41°60	18°58	91°28	44°47
7	26°66	5°20	1°49	41°63	18°95	90°69	44°34
9	25°96	6°44	2°13	41°66	19°31	90°09	44°20
11	25°24	7°69	2°77	41°69	19°66	89°49	44°06
13	24°50	8°95	- 3°41	+41°72	-20°00	88°88	43°91

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Greenwich Mean. Noon.	☉	Light- time.	Appar. Diam.	Defect of Illumi- nation.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24 ^h +. m
1905.		m	"	"		h m	
July 31		7 ^h 02 ^m 5	11 ["] 01	1 ["] 51	351°44	0 35 ^m 17*	
Aug. 2		7 ^h 120	10 ["] 86	1 ["] 51	332°34	1 53 ^m 66	
4		7 ^h 216	10 ["] 72	1 ["] 50	313°22	3 12 ^m 23	
6		7 ^h 312	10 ["] 58	1 ["] 50	294°09	4 30 ^m 89	
8		7 ^h 408	10 ["] 44	1 ["] 49	274°92	5 49 ^m 63	
10	194°28	7 ^h 504	10 ["] 31	1 ["] 48	255°74	7 8 ^m 46	39 ^m 42
12		7 ^h 600	10 ["] 18	1 ["] 47	236°55	8 27 ^m 33	
14		7 ^h 696	10 ["] 05	1 ["] 47	217°34	9 46 ^m 28	
16		7 ^h 792	9 ["] 93	1 ["] 46	198°12	11 5 ^m 27	
18		7 ^h 887	9 ["] 80	1 ["] 45	178°87	12 24 ^m 40	
20		7 ^h 984	9 ["] 69	1 ["] 43	159°62	13 43 ^m 53	
22	201°40	8 ^h 079	9 ["] 57	1 ["] 42	140°36	15 2 ^m 70	39 ^m 59
24		8 ^h 175	9 ["] 46	1 ["] 41	121°09	16 21 ^m 91	
26		8 ^h 271	9 ["] 35	1 ["] 39	101°80	17 41 ^m 21	
28		8 ^h 367	9 ["] 24	1 ["] 38	82°50	19 0 ^m 55	
30		8 ^h 462	9 ["] 14	1 ["] 37	63°19	20 19 ^m 93	
Sept. 1		8 ^h 557	9 ["] 04	1 ["] 36	43°87	21 39 ^m 36	
3	208°63	8 ^h 653	8 ["] 94	1 ["] 34	24°54	22 58 ^m 84	39 ^m 73
5		8 ^h 748	8 ["] 84	1 ["] 33	5°20	24 18 ^m 36	
7		8 ^h 843	8 ["] 74	1 ["] 31	345°85	0 58 ^m 16*	
9		8 ^h 939	8 ["] 65	1 ["] 29	326°49	2 17 ^m 74	
11		9 ^h 033	8 ["] 56	1 ["] 28	307°12	3 37 ^m 36	
13		9 ^h 128	8 ["] 47	1 ["] 26	287°74	4 57 ^m 03	
15	215°98	9 ^h 223	8 ["] 39	1 ["] 25	268°36	6 16 ^m 71	39 ^m 83
17		9 ^h 317	8 ["] 30	1 ["] 23	248°96	7 36 ^m 46	
19		9 ^h 414	8 ["] 22	1 ["] 22	229°55	8 56 ^m 26	
21		9 ^h 506	8 ["] 13	1 ["] 20	210°14	10 16 ^m 06	
23		9 ^h 600	8 ["] 05	1 ["] 19	190°73	11 35 ^m 86	
25		9 ^h 695	7 ["] 98	1 ["] 17	171°31	12 55 ^m 70	
27	223°42	9 ^h 790	7 ["] 90	1 ["] 15	151°88	14 15 ^m 59	39 ^m 94
29		9 ^h 885	7 ["] 82	1 ["] 14	132°44	15 35 ^m 52	
Oct. 1		9 ^h 979	7 ["] 75	1 ["] 12	112°99	16 55 ^m 49	
3		10 ^h 074	7 ["] 68	1 ["] 11	93°53	18 15 ^m 50	
5		10 ^h 169	7 ["] 61	1 ["] 09	74°07	19 35 ^m 51	
7		10 ^h 263	7 ["] 54	1 ["] 07	54°59	20 55 ^m 61	
9	230°93	10 ^h 357	7 ["] 47	1 ["] 06	35°11	22 15 ^m 71	40 ^m 05
11		10 ^h 451	7 ["] 40	1 ["] 04	15°62	23 35 ^m 85	
13		10 ^h 546	7 ["] 33	1 ["] 02	356°12	0 15 ^m 95*	

0 0

Greenwich Mean Noon.	P.	L-O.	B.	A-L.	B.	Q.	d.
1905.							
Oct. 15	23°74	10°22	- 4°05	+ 41°75	- 20°34	88°27	43°75
17	22°96	11°50	4°69	41°78	20°67	87°66	43°59
19	22°18	12°78	5°33	41°80	21°00	87°05	43°43
21	21°39	14°07	5°97	41°82	21°30	86°44	43°26
23	20°58	15°37	6°61	41°84	21°59	85°83	43°08
25	19°75	16°68	7°25	41°85	21°87	85°22	42°89
27	18°92	18°00	7°89	41°86	22°15	84°61	42°70
29	18°07	19°33	8°52	41°87	22°42	84°01	42°50
31	17°20	20°67	9°15	41°88	22°69	83°41	42°30
Nov. 2	16°33	22°02	9°78	41°88	22°93	82°81	42°10
4	15°45	23°38	10°40	41°88	23°16	82°22	41°89
6	14°56	24°75	11°02	41°87	23°38	81°65	41°68
8	13°65	26°13	11°63	41°86	23°59	81°08	41°46
10	12°74	27°52	12°24	41°84	23°78	80°52	41°24
12	11°82	28°92	12°84	41°81	23°96	79°96	41°02
14	10°90	30°32	13°44	41°78	24°13	79°41	40°80
16	9°97	31°74	14°03	41°74	24°29	78°87	40°57
18	9°03	33°17	14°61	41°69	24°44	78°33	40°34
20	8°08	34°61	15°18	41°63	24°57	77°79	40°11
22	7°13	36°06	15°74	41°57	24°69	77°26	39°87
24	6°17	37°52	16°29	41°50	24°80	76°75	39°63
26	5°21	39°00	16°83	41°41	24°90	76°25	39°38
28	4°24	40°49	17°36	41°31	24°98	75°76	39°13
30	3°27	41°99	17°88	41°21	25°05	75°29	38°88
Dec. 2	2°30	43°50	18°40	41°10	25°11	74°83	38°62
4	1°32	45°02	18°90	40°97	25°16	74°38	38°36
6	0°34	46°55	19°38	40°83	25°19	73°94	38°10
8	359°36	48°08	19°85	40°68	25°20	73°51	37°83
10	358°38	49°63	20°31	40°52	25°21	73°09	37°56
12	357°40	51°19	20°75	40°35	25°20	72°68	37°29
14	356°41	52°76	21°18	40°17	25°18	72°29	37°01
16	355°43	54°34	21°59	39°97	25°14	71°91	36°74
18	354°45	55°93	21°99	39°76	25°10	71°54	36°46
20	353°47	57°53	22°37	39°54	25°04	71°19	36°18
22	352°49	59°14	22°73	39°30	24°97	70°85	35°90
24	351°51	60°75	23°08	39°05	24°88	70°53	35°61
26	350°54	62°37	23°42	38°78	24°78	70°23	35°32
28	349°57	64°00	- 23°74	+ 38°50	- 24°68	69°93	35°02

Mar. 1904.

Observations of Mars, 1904-6.

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Greenwich Mean. Noon.	☉	Light- time.	Appar. Diam.	Defect of Illumi- nation.	Central Meridian.	Passage of Zero Meridian.	Interval between Passages 24 ^h + m
1905.	°	m	"	"		h m	
Oct. 15		10 ^h 64.1	7 ["] 27	1 ["] 01	336° 61	1 36.16	
17		10 ^h 73.6	7 ["] 20	0.99	317° 10	2 56.38	
19		10 ^h 83.0	7 ["] 14	.98	297° 58	4 16.64	
21	238° 50	10 ^h 92.6	7 ["] 08	.96	278° 05	5 36.95	40° 15
23		11 ^h 02.1	7 ["] 02	.95	258° 51	6 57.31	
25		11 ^h 11.6	6 ["] 96	.93	238° 96	8 17.70	
27		11 ^h 21.2	6 ["] 90	.92	219° 40	9 38.13	
29		11 ^h 30.8	6 ["] 84	.90	199° 83	10 58.60	
31		11 ^h 40.4	6 ["] 78	.88	180° 25	12 19.13	
Nov. 2	246° 10	11 ^h 50.0	6 ["] 72	.87	160° 66	13 39.69	40° 28
4		11 ^h 59.6	6 ["] 67	.85	141° 06	15 0.30	
6		11 ^h 69.2	6 ["] 62	.84	121° 45	16 20.94	
8		11 ^h 78.8	6 ["] 57	.82	101° 83	17 41.63	
10		11 ^h 88.4	6 ["] 51	.80	82° 20	19 2.36	
12		11 ^h 98.1	6 ["] 46	.79	62° 56	20 23.14	
14	253° 71	12 ^h 07.8	6 ["] 41	.78	42° 91	21 43.96	40° 39
16		12 ^h 17.5	6 ["] 35	.76	23° 26	23 4.78	
18		12 ^h 27.2	6 ["] 30	.75	3° 59	24 25.68	
20		12 ^h 37.0	6 ["] 25	.74	343° 91	1 6.17*	
22		12 ^h 46.8	6 ["] 20	.72	324° 22	2 27.15	
24		12 ^h 56.6	6 ["] 15	.71	304° 52	3 48.17	
26	261° 31	12 ^h 66.4	6 ["] 10	.69	284° 80	5 9.27	40° 55
28		12 ^h 76.3	6 ["] 06	.68	265° 07	6 30.42	
30		12 ^h 86.2	6 ["] 01	.67	245° 33	7 51.61	
Dec. 2		12 ^h 96.2	5 ["] 97	.65	225° 58	9 12.84	
4		13 ^h 06.1	5 ["] 92	.64	205° 82	10 34.11	
6		13 ^h 16.1	5 ["] 88	.63	186° 06	11 55.38	
8	268° 88	13 ^h 26.0	5 ["] 83	.61	166° 28	13 16.74	40° 68
10		13 ^h 36.0	5 ["] 79	.60	146° 49	14 38.14	
12		13 ^h 46.0	5 ["] 74	.59	126° 69	15 59.59	
14		13 ^h 56.1	5 ["] 70	.57	106° 88	17 21.17	
16		13 ^h 66.2	5 ["] 65	.56	87° 05	18 42.74	
18		13 ^h 76.3	5 ["] 61	.55	67° 22	20 4.31	
20	276° 40	13 ^h 86.4	5 ["] 57	.54	47° 38	21 25.92	40° 79
22		13 ^h 96.6	5 ["] 53	.52	27° 54	22 47.53	
24		14 ^h 06.7	5 ["] 49	.51	7° 69	24 9.20	
26		14 ^h 16.9	5 ["] 45	.50	347° 82	0 50.10*	
28		14 ^h 27.1	5 ["] 41	0.49	327° 94	2 11.88	

Greenwich Mean Noon.	P.	L—O.	B.	Λ —L.	B.	Q.	d.
1905. Dec. 30	348°61	65°65	—24°04	+38°21	—24°56	69°65	34°72
1906. Jan. 1	347°65	67°31	24°31	37°91	24°43	69°38	34°43
3	346°70	68°97	24°56	37°60	24°29	69°13	34°14
5	345°75	70°63	24°80	37°28	24°14	68°89	33°84
7	344°81	72°30	25°03	36°95	23°98	68°67	33°54
9	343°88	73°97	25°24	36°60	23°80	68°47	33°23
11	342°96	75°65	25°42	36°24	23°61	68°27	32°92
13	342°04	77°33	25°58	35°86	23°41	68°08	32°61
15	341°13	79°01	25°72	35°47	23°20	67°91	32°30
17	340°24	80°70	25°84	35°08	22°98	67°76	31°99
19	339°37	82°39	—25°94	+34°69	—22°75	67°62	31°68
Dec. 11	36°83	304°43	+20°77	—30°24	+25°16	292°06	28°13
13	37°08	305°70	20°44	30°50	25°12	291°82	28°45
15	37°32	306°96	20°10	30°76	25°08	291°57	28°77
17	37°54	308°22	19°75	31°02	25°03	291°31	29°10
19	37°75	309°46	19°39	31°27	24°98	291°05	29°42
21	37°93	310°70	19°03	31°52	24°93	290°77	29°74
23	38°10	311°93	18°66	31°76	24°86	290°49	30°06
25	38°25	313°16	18°28	32°00	24°79	290°20	30°39
27	38°38	314°38	17°89	32°23	24°71	289°90	30°71
29	38°48	315°59	17°49	32°46	24°63	289°59	31°04
31	38°57	316°79	+17°09	—32°69	+24°54	289°27	31°36

The position of *Mars*' north pole is assumed to be R.A. $317^{\circ} 17' 28$, N.P.D. $37^{\circ} 21' 12$ at the beginning of 1905, and R.A. $317^{\circ} 18' 20$, N.P.D. $37^{\circ} 20' 65$ at the end of 1906, these being the values deduced by Dr. Hermann Struve from observations of the satellites.

P denotes the position-angle of the axis of *Mars* ; L—O + 180° the right ascension of the Earth referred to the plane of *Mars*' equator, and reckoned from O, the point of the vernal equinox of *Mars*' northern hemisphere ; Λ —O + 180° the corresponding quantity for the Sun ; B B the declinations of the Earth and Sun reckoned from the plane of *Mars*' equator ; Q the position-angle of the greatest defect of illumination ; q the amount of the same ; d the areocentric angle between the Earth and the Sun.

Thus Λ —L is the angle between the meridians of *Mars*, which are central to the Earth and to the Sun ; the point where an image of the Sun should be looked for is distant $\frac{1}{2}(\Lambda - L)$ in longitude from the centre of the disc, on the side towards the fully illuminated limb, and is in areocentric latitude $\frac{1}{2}(B + B)$;

Greenwich Mean. Noon.	☉	Light- time.	Appar. Diam.	Defect of Illumi- nation.	Central Meridian.	Passage of Zero Meridian. h m	Interval between Passages 24 ^h +. m
1905. Dec. 30	0	14 ^m 374	5 ^{''} 38	0 ^{''} 48	308° 06	3 33 ^m 67	
1906. Jan. 1	283° 83	14 ^m 476	5 ^{''} 34	47	288° 17	4 55 ^m 49	40° 91
3		14 ^m 579	5 ^{''} 30	46	268° 28	6 17 ^m 32	
5		14 ^m 681	5 ^{''} 26	45	248° 37	7 39 ^m 22	
7		14 ^m 784	5 ^{''} 23	44	228° 46	9 11 ^m 13	
9		14 ^m 887	5 ^{''} 19	42	208° 54	10 23 ^m 07	
11		14 ^m 990	5 ^{''} 16	41	188° 62	11 45 ^m 02	
13	291° 18	15 ^m 093	5 ^{''} 12	40	168° 69	13 7 ^m 01	40° 99
15		15 ^m 196	5 ^{''} 09	39	148° 76	14 29 ^m 01	
17		15 ^m 299	5 ^{''} 05	38	128° 82	15 51 ^m 05	
19	294° 82	15 ^m 403	5 ^{''} 02	0 ^{''} 37	108° 88	17 13 ^m 09	41° 01
Dec. 11	93° 86	17 ^m 122	4 ^{''} 52	0 ^{''} 27	157° 26	13 53 ^m 47	39° 97
13		16 ^m 983	4 ^{''} 55	28	137° 81	15 13 ^m 42	
15		16 ^m 843	4 ^{''} 59	28	118° 37	16 33 ^m 33	
17		16 ^m 700	4 ^{''} 63	29	98° 93	17 53 ^m 23	
19		16 ^m 557	4 ^{''} 67	30	79° 50	19 13 ^m 10	
21	98° 32	16 ^m 412	4 ^{''} 71	31	60° 08	20 32 ^m 92	39° 90
23		16 ^m 267	4 ^{''} 75	32	40° 67	21 52 ^m 70	
25		16 ^m 120	4 ^{''} 79	33	21° 26	23 12 ^m 47	
27		15 ^m 972	4 ^{''} 84	34	1° 86	24 32 ^m 21	
29		15 ^m 822	4 ^{''} 88	35	342° 47	1 12 ^m 06*	
31	102° 80	15 ^m 672	4 ^{''} 93	0 ^{''} 36	323° 09	2 31 ^m 72	39° 83

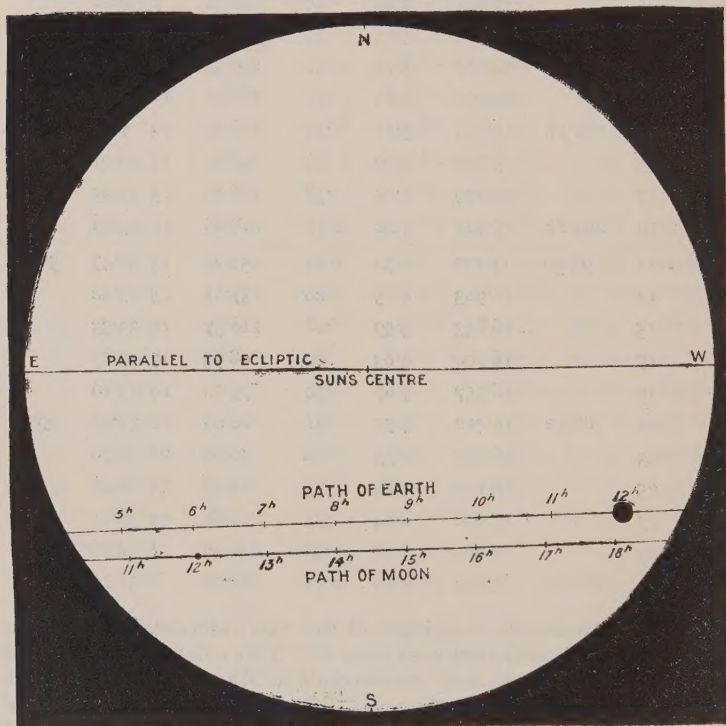
☉ is the areocentric longitude of the Sun referred to the plane of *Mars*' orbit and reckoned from O. This column indicates the season on *Mars*; ☉ = 0° corresponds to March 20 on the Earth, ☉ = 30° to April 20, and so on. The diameter of the planet at distance unity is assumed to be 9^{''} 30, the *Nautical Almanac* value being 9^{''} 36. The assumed time for light to travel over the unit distance is 8^m 31^s.

The zero meridian and period of rotation have been carried on unchanged from Mr. Marth's last ephemeris, the assumed sidereal rotation in 24^h of mean time being 350° 89' 20^s; the equivalent period of sidereal rotation is 24^h 37^m 22^s 654.

The passage of zero meridian is given for alternate rotations except those marked with an asterisk, which are separated from those preceding them by one rotation only. Where 24^h occurs in this column it is to be read as 0^h on the following day; e.g. 1904 Dec. 9^d 24^h is to be read as Dec. 10^d 0^h. The last column contains the interval between successive passages, and enables the intermediate passages to be readily deduced.

According to Mr. Lowell's observations the longitude of the central meridian should be increased by about 3° , and the times of transit of zero meridian diminished by about 13^m .

The ephemeris for 1906 December begins the following apparition of *Mars*, which is continued in the *Nautical Almanac* for 1907.



It is of interest to note that there will be a transit of the Earth and Moon across the Sun's disc as seen from *Mars* on 1905 May 8. The last two transits occurred 1879 November 12 and 1800 November 8 at the other node. The next will be in 1984 May. The following are approximate elements of the coming transit:

Semi-diameter of Sun seen from Mars = $10' 18'' \cdot 3$

„ „ Earth „ „ = $16 \cdot 2$

„ „ Moon „ „ = $4 \cdot 4$

Least distance of centres of $\odot \oplus$ = $4 \ 39$

	d	h	m
Ingress of Earth's centre	= May 8	4	10
Egress " "	=	8	12 52
Ingress of Moon's centre	=	8	10 17
Egress " "	=	8	18 48

The transits generally recur in a period of 284 years, there being four transits, two at each node, in this period; this is analogous to the 243-year period of recurrence of transits of *Venus* (which likewise generally contains four transits), and the 217-year period for transits of *Mercury*.

Thus we obtain the following years for May transits of the Earth: 1621, 1700, 1905, 1984, 2189, 2268, &c. And the following for November transits: 1516, 1595, 1800, 1879, 2084, 2163.

The transit of 1879 was discussed by Mr. Marth (*Monthly Notices*, vol. xxxix. p. 513). I think, however, that through some slip he has made the tracks of both Earth and Moon 1' too low on the Sun's disc.

Owing to the absolute symmetry of the illumination round the limb of the planet during the transit it will be a favourable opportunity for measuring the diameter and the polar compression.

The following are the dates of the commencements of the Martian seasons:

	d
Summer Solstice of N. Hemisphere 1905 Jan.	14.3
Autumnal Equinox " " " " "	July 16.3
Winter Solstice " " " " "	Dec. 9.8
Summer Solstice " " " " "	1906 Dec. 2.3

Benvenue, 55 *Ulundi Road*, *Blackheath*, *S.E.*:
1904 *March* 9.

Errata in Cape Double Star Results

Pages 131, 132, 133, column 5, for 1902 read 1903.

Page 134, line 3, for distance decreasing read distance increasing.

Erratum in Professor Turner's Paper on finding time of Sunset.

Page 194, line 11, for $l + \theta$ read $l + \theta + \text{const.}$

Erratum in Plate 14 to Mr. Ellis's paper on Aurora and Magnetic Disturbance.

On right-hand side of plate, Regions of the "Catalog," for 1861-1877 read 1761-1877.